Comment on “Constraints on sterile neutrinos in the MeV to GeV mass range”

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If the electron neutrino ν_e had an admixture with a massive sterile neutrino ν_h, the final state lepton phase space for β decay would be reduced differently for different available total energy release [1, 2]. In their comprehensive review, Bryman and Schrock [3] show a constraint from the $0^+ \rightarrow 0^+$ β decay rates on the admixture of ν_h with ν_e that is constant for ν_h masses from 1 to 10 MeV, while expressing concern that not all measurements may be accurate enough to support such a constant constraint. By updating the complete fitting approach of Deutsch *et al.* [1] to the most recent full $\mathcal{F}t$ value compilation of 2015, this Comment quantifies this concern: a constant constraint is not supported by the present quality of the measurements. Further, the constraints from the method of Deutsch *et al.* do not change much with the more recent data set, so could continue to be used by the community.

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In their comprehensive review of massive sterile neutrinos, Bryman and Schrock [3] show a potentially powerful constraint from the $0^+ \rightarrow 0^+$ decay values. They use the results of two 1990 analyses of heavy sterile neutrino ν_h admixtures [1, 2], scaling those results by the reduction in the $|V_{ud}|^2$ uncertainty determined by improvements in the compiled average $\mathcal{F}t$ value. They do discuss potential limitations on the data sets, particularly whether decays at all possible energy release have achieved sufficient accuracy. Nevertheless, the constraint plotted as “BD2” in Fig. 1 of Bryman and Schrock [3] is independent of the mass of the sterile ν_h between 1 and 10 MeV. Now, the main effect of such a neutrino on these decays is that for a given fixed sterile neutrino mass m_h, the phase space for lepton emission depends on the energy released in the various decays. For the constraint to remain the same at low and high m_h, the $\mathcal{F}t$ values at the extremes of energy release would actually need to have considerably higher accuracy than the others. This is not a feature of presently achieved measurements. This Comment inelegantly quantifies the issues by simply updating the complete approach of Deutsch et al [1] to the most recent full $\mathcal{F}t$ value compilation of 2015 [4].

This Comment uses the formalism and procedures of the heavy ν_h ansatz used in Ref. [1] and references therein, labelled “BD1” in Ref. [3] Fig. 1. The implementation of this Comment accurately reproduces the results of Deutsch *et al.* for the 1990 data set [1]. A sketch of the method appears here. For a given hypothetical sterile ν_h mass m_h, the measured $\mathcal{F}t_j$ for a given decay (denoted by index j) in terms of its admixture U^2 with ν_e is given by

$$\mathcal{F}t_j = \overline{\mathcal{F}t}[(1-U^2) + U^2 f_j(Q_j, Z_j, m_h)/f_j(Q_j, Z_j, 0)]$$

where that single U^2 and an average $\overline{\mathcal{F}t}$ are parameters to be separately fit to the ensemble of $\mathcal{F}t_j$'s. The lepton phase space integral $f_j(Q_j, Z_j, m_h)$ is determined by energy release Q_j and atomic number Z_j, and changes for each m_h. The phase space integrand is $F(Z, E_e) p_e E_e p_\nu E_\nu \; dE_e$, where energy conservation determines $E_\nu = Q + m_\beta - E_e$, there is dependence on m_h through $p_\nu^2 = E_\nu^2 - m_h^2$, and of course the integral terminates at smaller E_β for finite m_h. For this purpose a simple Fermi function $F(Z, E_e)$ turns out to be adequate numerically. It's worth noting this equation has the correct intuitive limits. For decays with $Q < m_h$, $f_j(Q_j, Z_j, m_h)$ vanishes, so the computed $\mathcal{F}t$ acquires a deficit by U^2. For decays with $Q \gg m_h$, then $f_j(Q_j, Z_j, m_h)$ is restored to its full value $f_j(Q_j, Z_j, 0)$, so the existence of m_h does not affect the computed value for $\mathcal{F}t_i$.

This update uses the $\mathcal{F}t$ compilation of Ref. [4]. A full and more accurate evaluation of more recent results Ref. [5] will be left to the expert compilers. Subtleties include experimental progress in determining uncertainties of isospin-breaking theory corrections [6], and individual and common uncertainties in radiative corrections [4] treated only approximately here.

An example fit for best U^2 is shown in Fig. 1 for m_h=2.2 MeV, where sensitivity is highest. The best fit is for a physically non-allowed negative U^2 with one σ significance (see Fig. 2 for the fit U^2 for each considered m_h. The 1990 data set used in Ref. [1] produced the same unphysical sign, but at 2σ significance, all tabulated there. Partly because of this unphysical sign, even though the 2015 compilation has considerably better average accuracy than the 1990 compilation, and the $\mathcal{F}t$ set agrees better with CVC, the two compilations set nearly equal contraints on m_h.

Fits are also shown in Fig. 1 for the best U^2 for m_h of 0.4 MeV and 5.0 MeV. It should be clear to produce similar effects compared to the best fit for m_h=2.2 MeV, the values of U^2 (see Fig. 2) must be much larger. These examples illustrate how little the lepton phase space changes at these extreme m_h, and indicate how much better the decays at low and at high Q would need to be to produce similar sensitivity to the low and high m_h.

The centroid results for U^2 are shown in Fig. 2 with 1 σ uncertainties. The 90% CL is plotted, using the statistical procedure adopted in Ref. [1] (taking the fraction of the distribution in the physically allowed region). The tightest constraints on U^2, near m_h=2.2 MeV, are

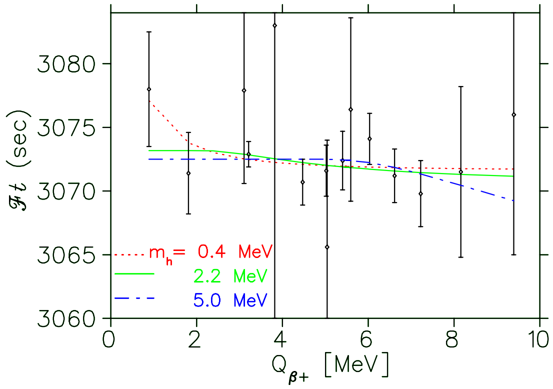

FIG. 1: Example fit of the method of Ref. [1] to the $\mathcal{F}t$ values compiled in Ref. [4] for the best U^2 for fixed m_h=2.2 MeV, where admixture sensitivity is greatest. The 2015 data support an unphysical negative admixture, though with less significance than the 1990 data used in Ref. [1]. Fits are shown for two further values of m_h, to which the data is much less sensitive.

almost the same as those in Ref. [1], and are about a factor of two less restrictive than Ref. [3] Fig. 1 "BD2." With the phase space dependence taken into account, the constraints at the higher and the very lowest m_h are naturally much poorer than at m_h=2.2 MeV.

Unlike Deutsch *et al.* [1], one could alternately constrain U^2 to be greater than zero for each fit with given m_h. In the present data set, that leads to U^2 converging at a shallower positive local minimum with with poor confidence levels of a few percent. The resulting 90% confidence limits, which are considerably less restrictive, are also plotted in Fig. 2.

Fig. 2 further demonstrates that allowing admixtures of m_h changes both the centroid and the uncertainty in the extracted average $\mathcal{F}t$. Since V_{ud} is a fundamental constant that must be known for many other physics purposes, this motivates direct searches for ν_h with competitive sensitivity.

Of course, instead of allowing $\overline{\mathcal{F}t}$ to float as in Deutsch *et al.* [1], one could fix it to the value predicted by CKM unitarity, while allowing the possibility of finite U^2. Such an ansatz is not relevant to the "BD2" contraint from Bryman and Schrock [3], who state that they are using the uncertainty achieved for V_{ud}, but not the center value. The present CKM unitarity deficit can then be accounted for by a sterile ν_h with mass m_h between about 4 and 60 MeV. The resulting admixture U^2 would be given directly by the CKM unitarity deficit of about 1x10^{-3}, with significance between 2 and 3 σ. The actual admixture would require evaluation of consistent calculations of a new radiative correction contribution with varying precision [9–11], well beyond the scope of this Comment and the ken of its author. Above 4 MeV, the lepton phase space is small enough that the $\mathcal{F}t$ values have inadequate sensitivity to constrain such an admixture. Below 3 MeV there are strong direct constraints from ^{20}F β decay [1]. Other powerful constraints on m_h from 3-60 MeV are reviewed by Bryman and Schrock [3] and also by the Borexino collaboration [8], many of which potentially eliminate such m_h if it were the sole new physics source– yet each of these less direct constraints could be relaxed by allowing multiple sources of new physics or other model dependence. Since such a ν_h would appear to be ruled out more directly from 3-10 MeV by the Bryman and Schrock "BD2" constraint, it could be important to the community to recognize that "BD2" is not supported by the data, the main point of this Comment.

Although everything in this Comment is based on published formalism, experiments, and compilations of other researchers, these small observations could motivate experimentalists searching directly for these m_h.

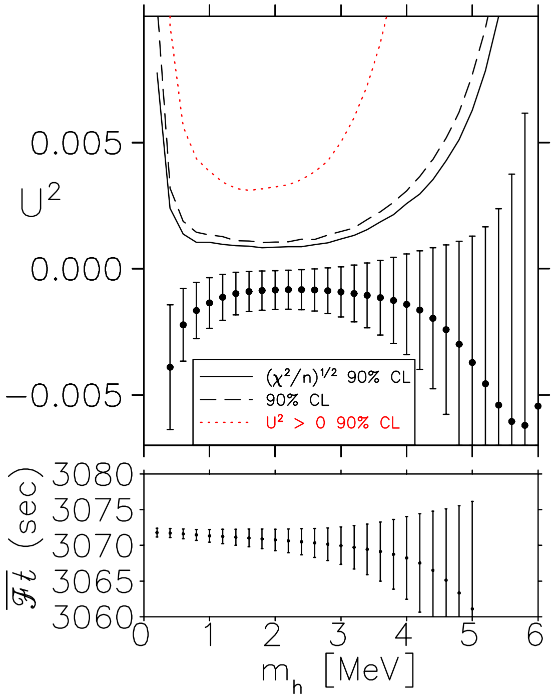

FIG. 2: Top: The extracted U^2 as a function of m_h with 1 σ uncertainties, and the resulting 90% CL (solid line), applying the methods of Ref. [1] to the 2015 $\mathcal{F}t$ values from Ref. [4]. The tightest 90% CL is 8.3×10^{-4}. The uncertainties shown approximately account for known partly common systematics [4] by reducing all $\mathcal{F}t$ uncertainties by $\sqrt{\chi^2/N}$. The dashed line shows the 90% CL without reducing uncertainties, with tightest constraint 1.0×10^{-3}. The dotted red line is the 90% CL when each U^2 is forced to be greater than zero for each fit to m_h , with tightest constraint 3.1×10^{-3}. See text. Bottom: The resulting changes in the fit average $\overline{\mathcal{F}t}$ values as a function of m_h, with 1 σ uncertainties.

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