Weak Interactions and Nuclei

- β decay basics
  Lepton long-λ expansion, Fermi function
  Allowed,Forbidden Decay; Selection Rules
- Weak quark eigenstates, CKM matrix
- Why the weak interaction is weak (at low energies)
- Quark-lepton interaction+QCD induces nucleon-lepton interaction terms:
  Conserved Vector Current,
  Partially (Poorly?) conserved Axial Current
- $\not{P}$ (complete) : lepton helicity
  Decay correlations

Further Symmetries:

- $\not{T}$ (tiny)
  CKM phase
  Atomic Electric Dipole Moments from:
  Nuclear Schiff, magnetic quadrupole,
  and from QCD Lagrangian
  Nuclear level spacing: Wigner distribution
  Nuclear reactions
- Weak neutral current examples:
  Weak interaction between nucleons
- $0\nu\beta\beta$ decay intro

Refs.: Wong 5.5-5.6;
Commins and Bucksbaum "Weak Interactions of Leptons and Quarks" and
Commins "Weak Interactions (Physics) 1st edition."
Commins' Notes Ph 250 UCB 1996 (see Canvas "Lecture Notes")
N. A. Jelley "Fundamentals of nuclear physics" Cambridge U.P.

β decay: Energy release, other basics, orbital angular momentum

$Q_{\beta-}$ = M(Z,N) - M(Z+1,N-1) using atomic masses
(this is in some sense accidental: the β is created in the nucleus and leaves; if nothing else happens, this would create a negative atomic ion...)

Q_{EC} = M(Z,N) - M(Z-1,N-1) - $|B.E.(\text{electron})|$

$Q_{\beta+}$ = M(Z,N)-M(Z+1,N-1) + 2 m_e

Sometimes EC is allowed energetically when β^+ is not.
Atomic electron overlap with nucleus is greater as one goes heavier; EC $\sim$ 1% at Z$\sim$40 isotopes where β^+ is allowed, but can be 10's of % at Z=82
Ratio is given well by atomic wavefunctions, and has some sensitivity to the weak interaction nature (Brysk and Rose, Rev Mod Phys 30 (1958) 1169)

- **Q can vary from 18 keV (t to ^{3}He) to $>$ 10 MeV**
(m_β= 0.511 MeV, so β's can be relativistic or non-relativistic.)

- **electron DeBroglie wavelength: λ= h/p = 2π(197 MeV fm)/ $\sqrt{E^2 - m_e^2}$**
For kinetic energy 1 MeV, this is 870 fm, much larger than the nucleus.
So the long-wavelength expansion we're about to make is a good one.
Similarly, ℓ=rxp$\sim$ 0.005 $\hbar$ is typically small

Fermi's Golden Rule, applied to β Decay to get rates

For now write the transition probability

$$W = \frac{2\pi}{\hbar} |\langle \phi_f(\vec{r}) | H | \phi_i(\vec{r}) \rangle|^2 \rho(E_f)$$

The initial state is simply the parent nucleus at rest:

$$|\phi_i(\vec{r})\rangle = |J_i m_i \vec{r}\rangle$$

The final state consists of 3 particles. Ignoring for now Coulomb effect between the β and final nucleus, this is a product of 3 parts, with plane waves for the leptons:

$$|\phi_k(\vec{r})\rangle = \frac{1}{\sqrt{V}} e^{i\vec{k}_e \cdot \vec{r}} \frac{1}{\sqrt{V}} e^{i\vec{k}_\nu \cdot \vec{r}} |J_f m_f r'\rangle$$

The V's normalize the plane waves. Expand the plane waves in terms of spherical harmonics (we could do this for γ-rays, too: we're about to do a 'long-wavelength expansion'):

$$e^{i\vec{k}\cdot\vec{r}} = \sum_0^\infty \sqrt{4\pi(2\lambda+1)}\, i^\lambda j_\lambda(kr) Y_{\lambda 0}(\theta, 0)$$

where $\vec{k} = \vec{k}_e + \vec{k}_\nu$ and θ is the angle between $\vec{k}$ and $\vec{r}$.

Now we make the long-wavelength expansion:

$$j_\lambda(kr) \stackrel{kr<<1}{\rightarrow} \frac{(kr)^\lambda}{(2\lambda+1)!!}$$

so that the final state wavefunction becomes:

$$|\phi_k(\vec{r})\rangle = \frac{1}{V}\left(1 + i\sqrt{\frac{4\pi}{3}}(kr)Y_{10}(\theta,0) + O(k^2r^2)\right)|J_f m_f r'\rangle$$

Even without the formal weak interaction theory, we can now surmise the form of *H*, the nuclear part of the β-decay operator.
Neutrons are transformed into protons $\Rightarrow$ the nuclear operator:
1) must be one-body, i.e.only one nucleon is involved at a time;
2) must involve single particle isospin raising/lowering operator $\tau_\pm$ (this comes from the 'vector' V 'Fermi' part of 'V-A')

• **The axial vector 'A' 'Gamow-Teller' part produces a product of σ and $\tau_\pm$**
Then we can write the matrix element $\langle \phi_f(\vec{r})|H|\phi_i(\vec{r})\rangle =$

$$\frac{1}{V}\langle J_f m_f r|\sum_{j=1}^{A}(G_V\tau_\pm(j) + G_A\vec{\sigma}(j)\tau_\pm(j))\left(1 - i\sqrt{\frac{4\pi}{3}}(kr)Y_{10}(\theta,0) + O(k^2r^2)\right)|J_i m_i r'\rangle$$

• **The Fermi operator does nothing to space/spin. So it only links isobaric analog states, or pieces of isobaric analog states, i.e. states with same wavefuntion except proton/neutron interchange.**

• **This form shows both the allowed terms and some '1st forbidden' terms: these are from the same nuclear operators σ and τ, but including the next order of the lepton long wavelength expansion and thus suppressed. However, the nuclear matrix elements also vary, so some 1st forbidden rates are faster than some G-T.**

The 1st-forbidden operators all flip the nuclear parity, so don't contribute at all to the allowed transitions between states of same parity.

Density of final states

We have to make sure that momentum and energy are conserved properly among the 3-body final state.
We start by writing the ν density as a statistical mechanical result (and integrate over all angles for the time being):

$$dn_\nu = \frac{V}{2\pi^2\hbar^3} p_\nu^2 dp_\nu$$

$E_\nu^2 = m_\nu^2 + p_\nu^2$ but $m_\nu < 3$ eV $\approx$0 so E_ν=p_ν.
We can ignore the recoil energy for kinematics
(though keeping it produces corrections to collelations, 'recoil order terms' $\sim$ 0.01)
which gives the relation:
$E_\nu = Q - K_e$
where Q is the total kinetic energy released in the decay, and K_e is the kinetic energy of the electron. (This kinetic energy is sometimes written 'E' in the literature)
I'll also make use of maximum total e energy $E_0 = Q + m_e$ and $E_\nu = E_0 - E_e$

Density of charged lepton final states: Fermi function

The density of charged-lepton states gets perturbed in the presence of the nuclear Coulomb field, so (also integrating over all angles)

$$dn_e = \frac{V}{2\pi^2\hbar^3} F(Z, K_e) p_e^2 dp_e$$

$F(Z, K_e)$ is a correction factor called the 'Fermi' function.
If the lepton is nonrelativistic, and the nucleus is pointlike, you can take the probability of the lepton at the nucleus, given by the Coulomb wavefunction at the origin:

$$F(Z, K_e) = \left| \frac{x}{1 - e^{-x}} \right|$$

where $x = -1 \times \pm 2\pi\alpha Zc/v$ for $\beta^{\pm}$ decay, with $\alpha \approx 1/137$ the fine structure constant.
This one misses the total decay strength by 10% at Z=26 and Q=7 MeV.

Fermi function (continued)

Sometimes people use a relativistic expression, though since the relativistic density is ∞ at the nucleus, Fermi took it at the nuclear surface (deShalit and Feshbach eq. IX.2.15, Fermi Zeit. Physik 88 (1934) 161):

$$F(Z, K_e) = 2(2kr)^{2(s-1)} \frac{1+s}{s^2+\eta^2} \left| \frac{e^{\pi\eta/2}\Gamma(s+1+i\eta)}{\Gamma(2s+1)} \right|^2$$

using the Γ function and using η=x/(2π), and $s = \sqrt{1-(\alpha Z)^2}$

There are just-as-easy ones available around that do better.

To do the full job, you put in electron screening of the atom, and several other effects (like realistic atomic wavefunctions instead of just relativistic Coulomb functions for a point charge).

See Sir Denys Wilkinson's 5-part series in Nuclear Inst. and Meth.

Soon we reach an example of why the Fermi function is important:

β energy spectrum for allowed decay

Integrating $p_e^2 dp_e p_\nu^2 dp_\nu \delta(Q - K_e - E_\nu)$ over p_ν, ignoring recoil-order terms and forbidden decay (so the nuclear matrix elements have no spatial/momentum dependence),

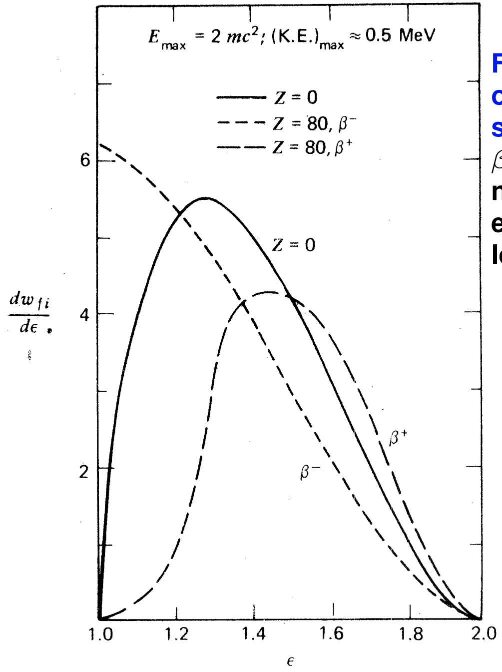
$$W(p_e)dp_e = \frac{1}{2\pi^3\hbar^3 c^3} \sum_{\mu\ m_f} |\langle J_f m_f r| O_{\lambda\mu}(\beta) | J_i m_i r' \rangle|^2 F(Z, K_e) p_e^2 (Q - K_e) \sqrt{(Q - K_e)^2 - m_\nu^2} dp_e$$

with $O_{\lambda\mu} = \sum_{j=1}^{A} (G_V \tau_\pm(j) + G_A \vec{\sigma}(j) \tau_\pm(j))$. Differentiating $E^2 = p^2 + m^2 \Rightarrow pdp = EdE$,

$$W(E_e)dE_e \propto F(Z, E_e) E_e p_e (E_0 - E_e) \sqrt{(E_0 - E_e)^2 - m_\nu^2} dE_e$$

- **The decay rate $\sim Q^5$, a large dependence. This is just from three powers of momentum for each lepton, minus one for energy conservation.**
- **The spectrum gets distorted at the very endpoint (large K_e, near Q), by the ν mass, which has upper limit (from $^3\mathrm{H}$ decay KATRIN) 1.1 eV at 90% confidence.**

(Most forbidden decay operators produce large changes in this energy spectrum)



Fermi function effect on β energy spectrum

β^- is 'pulled into' nucleus... a big effect for high Z and low E_β

ft value for allowed decay

After doing the phase space integration, we can write down the answer:

$$ft = \frac{K}{|M_F|^2 + g_A^2|M_{GT}|^2}$$

$$K = \frac{2\pi^3\hbar^7 \ln 2}{m_\beta^5 c^4 G_V^2} = 6142 \pm 3.2\mathrm{s}$$

If you include isospin mixing and 'radiative' corrections, you can define the quantity *Ft* that is actually constant for the Fermi transitions:

$$Ft = ft(1-\delta_C)(1+\delta_R) = \frac{K}{G_F^2|V_{ud}|^2|M_{fi}|^2(1+\Delta_R)}$$

where $|M_{fi}|^2$ = T(T+1)-$T_3(T_3+1)$ as we saw last time.
Isospin-breaking corrections δ_C are parameterized by two sources:
1) isospin mixing of with other 0^+ states
2) the spatial wavefunctions are slightly different because the protons repel each other 'radial mismatch'.

Superallowed Ft values

Consider Fermi beta decay in many 0^+ to 0^+ cases. We can sum over the nucleons

$$\sum_{k=1}^{A} \tau_{\pm}(k) = T_{\pm}$$

where $T_{\pm}$ lowers or raises the 3rd component of SU(2) isospin for the whole nucleus, just like the lowering and raising operators for SU(2) spin. The Fermi operator's matrix element is

$$\langle J_f M_f T_f T_{0f} | \sum_{k=1}^{A} \tau_{\pm} | J_i M_i T_i T_{0i} \rangle$$
$$= \sqrt{T_i(T_i+1) - T_{0i}(T_{0i} \pm 1)}$$

if $J_f = J_i,\ M_f = M_i,\ T_f = T_i$, and $T_{0f} = T_{0i} \pm 1$; 0 otherwise.

For these cases, the ft value then given by just some constants, which are given by the weak interaction strength. (f=integral over phase space). I.e. they all should have the same intrinsic strength.
The vector operator is related to the electric charge operator. We know electric charge is conserved. The “conserved vector current” hypothesis of Feynman and Gell-Mann: by analogy they theorized that the vector part of the weak interaction is also conserved. This eventually leads to electroweak unification. This has many consequences. For example, for the vector part of the weak interaction we can go straight from the quark matrix element to the nucleon one to the nucleus one.

Effect of different Fermi functions on superallowed Ft's

Z	Q	no Fermi	Non-rel	Fermi's	Towner '05	error
5	0.88577	3540.7	3005.8	3030.2	3073.0	4.9
7	1.80851	3618.7	2985.0	3028.2	3071.9	2.6
12	3.21071	3957.0	2905.2	3015.4	3072.9	1.5
16	4.46971	4252.2	2832.3	3010.9	3071.7	1.9
18	5.02234	4400.4	2786.2	3002.9	3072.2	2.1
20	5.40358	4548.1	2732.9	2991.7	3075.6	2.5
22	6.02863	4696.5	2679.4	2979.5	3078.5	2.4
24	6.61039	4846.2	2622.4	2966.1	3071.1	2.7
26	7.22056	5004.4	2566.7	2956.2	3071.2	2.8

Towner's include isospin mixing corrections.

Note Fermi's 1934 function isn't really good enough for this, while "Towner" includes 1% corrections from isospin mixing, or rather the difference in isospin mixing between the parent and daughter. These are parameterized by:

1) different isospin configurations mixed;

2) different wavefunctions because the nuclei have different radii.

IMME is fit mass-by-mass, adjusting an effect Coulomb interaction in a shell model.

(A technical check of neutron occupancies is used in the 2020 versions.)

These are all consistent: a test of CVC hypothesis

J.C. Hardy, I.S.Towner, Phys Rev C 102 044501 (2020)

J. C. HARDY AND I. S. TOWNER

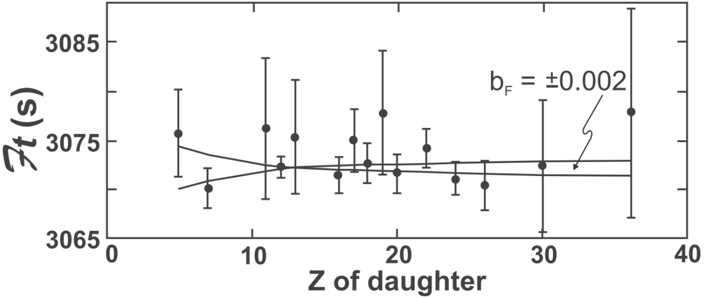


FIG. 7. Corrected $\mathcal{F}t$ values from Table XVI plotted as a function of the charge on the daughter nucleus, Z. The curved lines represent the approximate loci the $\mathcal{F}t$ values would follow if a scalar current existed with $b_F = \pm 0.002$.

Notice a constant goes through all– 5/15 should miss at 1 σ. There is a common systematic uncertainty from a radiative correction which is folded into each of these points.
This test is used to gain confidence in the isospin breaking calculations.
It is also the strongest constraint on a particular non-SM interaction, though there is known double-counting if you're using it to test the isospin breaking– hopefully more accurate calculations are possible.
The actual value quoted for V_{ud} sets $b_F = 0$; if b_F floats, V_{ud}'s uncertainty goes up.

Quark eigenstates in the weak interaction: Cabibbo angle

To explain some weak decays, in particular ratios of semileptonic baryon decays with and without strangeness,

the weak interaction mixes the *d* and *s* quarks, so you can think of the *u* changing to *d* in β decay as:

$|u\rangle \rightarrow |d\rangle + \epsilon|s\rangle$ **i.e.** $|u\rangle \rightarrow \cos(\theta_C)|d\rangle + \sin(\theta_C)|s\rangle$

θ_C, the Cabibbo angle, is a parameter whose value (13.04^o) is unexplained so far from underlying physics. (Like any mixing 'angle', the angle is in an abstract space, and it's just a simple way to normalize wavefunctions)

For 3 families of particles, this generalizes to

$\rightarrow$ 3x3 unitary "CKM" matrix between $|d\rangle, |s\rangle, |b\rangle$

From superallowed ft values we get a vital physics constant: V_{ud}

The quark eigenstates of the weak interaction are not the same as the mass eigenstates. They are related by a unitary transformation.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

As for any unitary matrix, top row has the property:
$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

The superallowed *Ft* value, compared to muon decay (the strength of the leptonic weak interaction), gives you V_{ud}. (V_{ub} is very small and does not matter.)

There's been a long struggle over V_{us}, which comes from kaon decays or hyperon β decay, with useful checks from theory with more than one possible solution.

CKM unitarity test is off by 2-3 σ at 0.1% from most recent reevaluations of radiative corrections (see Towner Hardy review below).

Again, each *Ft* value has an isospin mixing calculation done phenomenologically, because initial and final wavefunctions are not identical. The uncertainty and centroids of these calculations are still an open question.

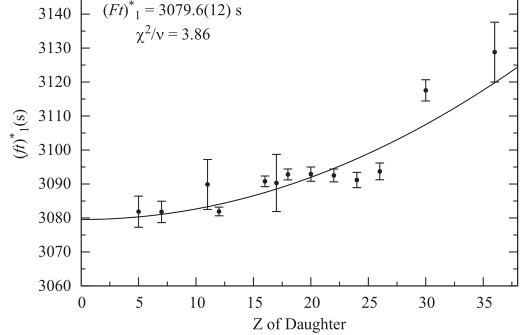
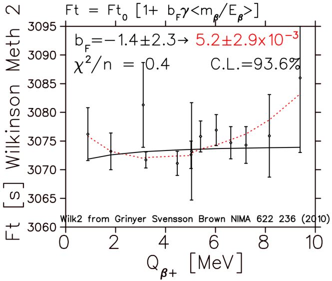


Fig. 2. Plot of the $(ft)_1^*$ data points that do not include theoretical corrections for isospin symmetry breaking and the resulting quadratic fit giving the global trend



Phenomenological estimate of isospin breaking uncertainty (and b_F)

- **Grinyer, Svensson, Brown NIMA 622 236 (2010)**

Using 'Wilkinson Method 2'
Correcting flucuations in *Ft* in each shell, yet allowing magnitude of isospin breaking to vary phenomenologically with Z^2.
V_{ud} changed by 0.2σ. σ_{Vud} increased by 1.3.

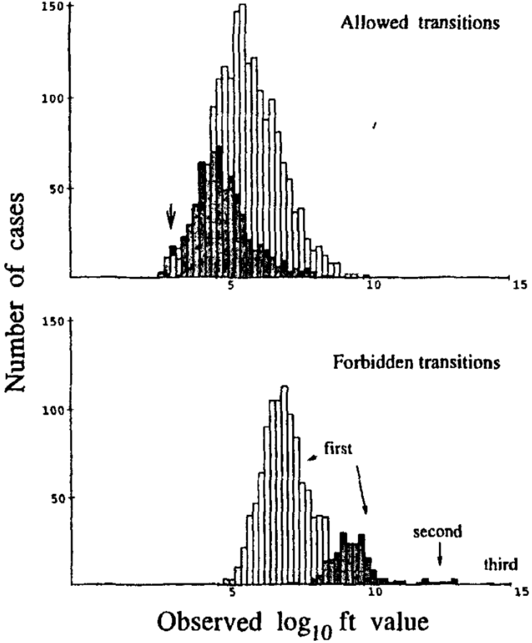
- **I, JB ☺, then floated b_{Fierz} and isospin breaking on compiled 2015 data set.**
- **If done on the 2020 data set (improved ^{14}O, ^{62}Ga, etc.) could better assign σ to the isospin breaking and the Fierz term, perhaps without much cost to precision in V_{ud}**

log(ft) for β decay

Wong Figure 5.8

As we said above, G-T transitions preserve nuclear π, while 1st-forbidden transitions flip nuclear π.

If the *ft* values are different enough, that can distinguish the transition and be used to determine π.
However, the *ft* values for G-T and 1st forbidden overlap.
Sometimes the nuclear matrix element for G-T decay is accidentally small.
(E.g. ^{14}C GT decay has log(ft) of 9.0, five orders slower rate than the fastest GT's)



Selection rules

Fermi

G-T

γ_5 dominates $0^- \to 0^+$

$\sigma \cdot r$ suppressed by r/λ

'1st forb. unique' $2^\pm \leftrightarrow 0^\mp$
One operator $\Rightarrow$ calculable
correlations from a.m.

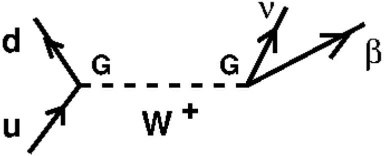
TABLE I. Allowed and first-forbidden nuclear matrix elements and their selection rules (K designates the rank of the transition operator, when regarded as a tensor).

	Matrix element	K	ΔJ	$\Delta\pi$
Allowed	$C_V\int 1$	0	0	+1
	$C_A\int \boldsymbol{\sigma}$	1	0, ±1 (no 0→0)	+1
First forbidden	$C_A\int \gamma_5$, $C_A\int(\boldsymbol{\sigma}\cdot\mathbf{r}/i)$	0	0	−1
	$C_V\int \mathbf{r}i$, $C_V\int \boldsymbol{\alpha}$, $C_A\int(\boldsymbol{\sigma}\times\mathbf{r})$	1	0, ±1 (no 0→0)	−1
	$C_A\int iB_{ij}$	2	0, ±1, ±2 (no 0→0, no 1→0, no 0→1)	−1

Weidenmüller Rev Mod Phys 33 574 (1961)

Why the Weak Interaction is weak at low energy

We've already developed the Yukawa potential using the Klein-Gordon equation.
The same physics can be looked at with the propagator in the Feynman diagram for $W^{\pm}$ exchange:
β decay is purely weak $\Rightarrow$ physics at scale M_W = 80GeV/c^2



Propagator+vertices: T $\propto \frac{G_S(-g^{\mu\nu}+p^\mu p^\nu/M_W^2)G_S}{p^2-M_W^2} \xrightarrow{p<<M_W} \frac{G_S^2}{M_W^2} \Rightarrow$
Rates [ignoring interference!] $\propto \frac{G_S^4}{M_W^4}$
So the massive W^+ makes the interaction strength small for β decay with $p \sim MeV$
At high $p \sim M_W$, the interaction has the same coupling constant as electromagnetism
For nucleons, G can be different from the quark-lepton couplings

Conserved Vector Current hypothesis with Dirac formalism

CVC is sometimes considered more for its consequences than for the physics behind it, so I'm going through the physics assumptions.

- **Construct the E&M current for pointlike particles and show its derivative is zero, simply because of conservation of electric charge.**
- **Consider what happens if the particles are composite, like nucleons. One gets some relations for 'form factors' describing the nucleons, relations necessary to keep this current conserved.**
- **Hypothesize that the vector part of the weak current should be similarly conserved, and show what that implies for weak interaction physics.**

I'll use Dirac formalism, because the currents are all relativistic. I'll cite the limited formalism I need as I go along.

The S.M. interaction has W exchange, which at momenta $<< M_W$ produces this quark-lepton current-current Langrangian density that is purely 'V-A' (using the opposite-signed convention):

$$L = \frac{G}{\sqrt{2}} J^{\mu} \bar{J}^{\dagger}_{\mu} + h.c. \text{ with } J_{\mu} = J^{(lep)}_{\mu} + J^{(had)}_{\mu} \text{ and } J^{(lep)}_{\mu} = \bar{\psi}_e \gamma_{\mu}(1+\gamma_5)\psi_{\text{neutrino}}$$

We would really like to just deal with quarks, so that we could write something like:

$$J^{(had)}_{\mu} = J^{quark}_{\mu} = \bar{\psi}_d \gamma_{\mu}(1+\gamma_5)\psi_u$$

because then everything would be automatically V-A, just like purely leptonic weak interactions (like μ decay).
However, we're stuck with nucleons, composite particles made of quarks. So QCD can 'induce' other terms as it combines quarks into the nucleon wf's.
So we have to go back and construct a general Lorentz vector for the hadrons (to make a bilinear covariant with the Lorentz vector of the leptons), along with an a Lorentz axial vector for the hadrons (to make a bilinear covariant with the axial current of the leptons).
First we'll back up and do this for the E&M current, which is purely vector. The fact that this vector current is conserved (electric charge is conserved) puts constraints on the composite terms:

First we consider the E&M current, take its divergence, and use Dirac equation $(\gamma_\mu \partial_\mu + m)\psi = (\not{p} + m)\psi = 0$

$J_\mu = -e\bar{\psi}\gamma_\mu\psi$ (more properly, matrix element $\langle p'|J_\mu(E\&M)|p\rangle$) for particle with momentum $p \to p'$)

using plane-wave solution to Dirac eq. $\psi = u(p)e^{ip\cdot x}$

$$\begin{aligned}\partial_\mu J_\mu/(-e) &= \\ \partial_\mu \left[\bar{u}(p_2)\gamma_\mu u(p_1)e^{i(p_1-p_2)\cdot x}\right] \\ &= [\bar{u}(p_2)(p_1 - p_2)_\mu \gamma_\mu u(p_1)]\, e^{i(p_1-p_2)\cdot x} \\ &= [\bar{u}(p_2)\ \not{p}_1 u(p_1) - \bar{u}(p_2)\ \not{p}_2 u(p_1)]\, e^{i(p_1-p_2)\cdot x} \\ &= i(m_1 - m_2)\bar{u}(p_2)u(p_1)e^{i(p_1-p_2)\cdot x} = 0\end{aligned}$$

because $m_1 = m_2$ in E+M interactions
So this E+M current is conserved, so charge is conserved, QED ☺

Electromagnetic current for composite particles

Before we go back to the weak interaction, it is instructive to write down a general electromagnetic current for a composite particle, take its divergence, and set that to zero. We will get a direct prediction about a corresponding term in β decay from CVC.
For composite particles like nucleons, we have to again write the most general Lorentz vector that can be constructed from γ_μ's and momenta, subject to:
a) momentum conservation means there are only two independent momenta, the difference $k_\mu = (p_2)_\mu - (p_1)_\mu$ and the average $K_\mu = 1/2(p_2 + p_1)_\mu$ of the individual momenta
b) not more than two γ matrices, because three γ's can be written as one γ with γ_5: I, γ_μ, and $\sigma_{\mu\nu} = \frac{1}{2i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$
c) use Dirac eq., i.e. replace $\not{p}_1$ with im_1 when adjacent to spinors.
Then the most general forms of Lorentz vectors are: γ_μ, $\sigma_{\mu\nu}k_\nu$, k_ν, $\sigma_{\mu\nu}K_\nu$, K_ν.
It turns out that the matrix elements of the last two can be rewritten in terms of the other 3. (Perhaps just reflecting that in the CM frame the total momentum is zero.)

So we can write a general form for the E&M current of a composite particle:

$$J_\mu/e = \bar{\psi}\left[F_1\gamma_\mu - \frac{F_2}{2m}\sigma_{\mu\nu}k_\nu + iF_3k_\mu\right]\psi$$

where F_1, F_2, F_3 are form factors, scalar functions of k^2. Is this conserved?

$$-\partial_\mu J_\mu/e = u(\bar{p}_2)\left[F_1 \not{k} - \frac{F_2}{2m}\sigma_{\mu\nu}k_\nu k_\mu + iF_3k^2\right]u(p_1)e^{ik\cdot x}$$

The 1st term vanishes as above
The 2nd term is zero independent of F_2, because $\sigma_{\mu\nu}$ is completely antisymmetric.

$$\sum_{\mu\nu}\sigma_{\mu\nu}k_\nu k_\mu \equiv \sum_{\mu<\nu}\left[\sigma_{\mu\nu}k_\nu k_\mu + \sigma_{\nu\mu}k_\mu k_\nu\right] = \sum_{\mu<\nu}\left[\sigma_{\mu\nu} + \sigma_{\nu\mu}\right]k_\mu k_\nu = 0$$

The third term is <u>not</u> zero, so for CVC to hold, $F_3(k^2)$=0.
The 2nd term can be related to the magnetic moments, in particular the non-Dirac 'anomalous' magnetic moments, so:
For the proton, $F_1^p(0)$=1, $F_2(p)=\mu_p$-1 = 1.793
For the neutron, $F_1^n(0)$=0, $F_2(n)=\mu_n$ =-1.913

Formally setting up isospin-changing operators for a weak 'current':

Recall results from angular momentum algebra: define isospin raising/lowering operators

$T_{\pm} = T_1 \pm iT_2$

$T_{\pm}|T, T_z\rangle = \sqrt{T(T+1) - T_z(T_z \pm 1)}|T, T_z \pm 1\rangle$

For spin-1/2, $T_z|1/2, \pm 1/2\rangle = \pm 1/2|1/2, \pm 1/2\rangle$

$T_+|1/2, -1/2\rangle = |1/2, 1/2\rangle$

Now write the E&M vertex function in terms of isoscalar and isovector parts:

$$e\left(\frac{1}{2}\left[F_1^S\gamma_\mu - \frac{F_2^S}{2m}\sigma_{\mu\nu}k_\nu\right] + \left[F_1^V\gamma_\mu - \frac{F_2^V}{2m}\sigma_{\mu\nu}k_\nu\right]T_z\right)$$

$$F_1^S = F_1^{(p)} + F_1^{(n)} = 1 + 0 = 1$$

$$F_1^V = F_1^{(p)} - F_1^{(n)} = 1 + 0 = 1$$

$$F_2^S = F_2^{(p)} + F_2^{(n)} = -0.120$$

$$F_2^V = F_2^{(p)} - F_2^{(n)} = +3.706$$

Now we can finally write the weak vertex function for the hadron part:

$$\frac{g}{2\sqrt{2}} V_{ud} \left[\left(g_V \gamma_\mu - \frac{g_M}{2m} \sigma_{\mu\nu} k_\nu + i g_S k_\mu \right) + \left(g_A \gamma_\mu - \frac{g_T}{2m} \sigma_{\mu\nu} k_\nu + i g_P k_\mu \right) \gamma_5 \right] T_\pm$$

where we have also included the similar axial vector terms, to form the covariant piece with the lepton axial vector current.

The CVC hypothesis includes some bold assertions:

a) Vector portion of weak current is conserved, analogous to E&M current

b) The two vector weak currents– the β^+ and β^- decay, given by the terms with $T_\pm$ isospin raising/lowering operators– and the isovector part of the electromagnetic current are members of an isotriplet of current operators

This implies:

i) $g_V = F_1^V$=1.00. Presence of strong interactions has left this term completely untouched $\Rightarrow$ unrenormalized. This has many physics consequences.

ii) $g_M = F_2^V = \mu_p - \mu_n - 1 = 3.70$

This term in the weak current of the nucleon is related to the anomalous magnetic moments of the nucleons, called “Weak magnetism”

iii) g_S = 0! The “induced scalar” term must be zero for CVC to hold

So now our full lepton-nucleon interaction density is (Morita Hyp. Int. 21 143 (1985)):

$$\sqrt{2}L = [V_\lambda + A_\lambda]\left[\bar{\psi}_e\gamma_\lambda(1+\gamma_5)\psi_\nu\right] + \left[V'_\lambda + A'_\lambda\right]\left[\bar{\psi}_\nu\gamma_\lambda(1+\gamma_5)\psi_e\right]$$

with explicitly different forms for $\beta^\pm$ decay:

$$V_\lambda = \bar{\psi}_p\left(g_V\gamma_\lambda + \frac{g_M}{2m}\sigma_{\lambda\rho}k_\rho + ig_Sk_\lambda\right)\psi_n \quad A_\lambda = \bar{\psi}_p\gamma_5\left(g_A\gamma_\lambda + \frac{g_T}{2m}\sigma_{\lambda\rho}k_\rho + ig_Pk_\lambda\right)\psi_n$$

$$V'_\lambda = \bar{\psi}_n\left(g_V^*\gamma_\lambda + \frac{g_M^*}{2m}\sigma_{\lambda\rho}k'_\rho - ig_S^*k'_\lambda\right)\psi_p \quad A'_\lambda = \bar{\psi}_n\gamma_5\left(g_A^*\gamma_\lambda - \frac{g_T^*}{2m}\sigma_{\lambda\rho}k'_\rho + ig_Pk'_\lambda\right)\psi_n$$

$$k = k_p - k_n = -k'$$

Yes, the hadron part, because of the QCD-driven "dressing" within the nucleon, is more complicated than the lepton part.
g_S and g_T terms change sign from electron to positron decay. These are therefore odd under charge symmetry. So they vanish in isobaric analog decays to the extent that charge symmetry is good. These are called "2nd-class currents" $\rightarrow$

There are 2 ways to make 2nd-class currents in a quark model:

- **Remembering Standard Model has $\bar{u}\gamma_\mu d$ and $\bar{u}\gamma_5 d$ terms only, add derivative terms like $\partial_\mu \bar{u}d$ and $\partial^\nu \bar{u}\sigma_{\mu\nu}\gamma_5 d$**

  These are not renormalizable, one large reason they were excluded from the Standard Model (Weinberg Phys. Rev. 112 1375 (1958)).

  [One perspective is that the Standard Model itself may be an Effective Field Theory good up to some very high energy. Naively, maybe that means renormalizability is not an exact logical requirement. However, deliberately introducing a manifestly unrenormalizable term would still be a very complicated move for the main part of one's basic theory.]
- **Introduce a new quantum number in addition to color and flavor! (Feynman famously called this q.n. 'smell'?). You can also interpret this as a second set of quarks (Holstein Treiman PRD 13 3059 (1976)) carrying this quantum number.**

  A related scenario: recently people consider extra sectors of particles not interacting much with us, but interacting strongly among themselves. QCD-like symmetries turn out to be a feasible way to generate dark matter. There are tight constraints from experiment on such scenarios.
- **The best experimental limits on 2nd-class currents are from direct dedicated β decay measurements, which allow 2nd-class current effects about an order of magnitude larger than the known ones from charge-symmetry breaking in QCD.**

Formal extension from nucleons to nuclei The hadron current we have written is for the spin-1/2 nucleon, where the μ is the only non-Dirac electromagnetic moment.

If you are describing nuclei (or hadrons) with spin $> 1/2$, then higher-rank electromagnetic moments also, by CVC, contribute to the weak vector current.
E.g., the electric quadrupole moment produces a component in the weak vector current.
Similarly, additional nuclear-structure dependent form factors appear for $J > 1$ in the axial vector current.

Holstein generalizes from nucleons to nuclei and writes decay correlations: Rev. Mod. Phys. 46 789 (1974) erratum 48 673; or “Weak Interactions in Nuclei”.
Nuclei are treated as “elementary particles” and form factors are introduced to include moments and effects from their nonpointlike size. Finite-size effects are
In isobaric analog decays, the vector current part is given by the measured electromagnetic moments. The g_T term in isobaric analog decays is zero, but in pure Gamow-Teller decay it is not zero, producing a part that depends on a nuclear structure calculation whose accuracy can limit the sensitivity to new physics.
Holstein’s approach considers ‘recoil-order’ terms $\sim (E_\beta/M)^N$ for N=1,2,3. Convergence is not guaranteed of such a series.

Behrens&Bühring “Electron Wavefunctions and Nuclear β Decay” has forbidden β decay

Finite nuclear S.M. expressions gain complexity with those corrections

$$l^\mu\langle\beta|V_\mu|\alpha\rangle = \delta_{JJ'}\delta_{MM'}\left(a(q^2)\frac{P\cdot l}{2M} + e(q^2)\frac{q\cdot l}{2M}\right) + ib(q^2)\frac{1}{2M}C^{M'k;M}_{J'1;J}(\mathbf{q}\times\mathbf{l})_k$$
$$+ C^{M'k;M}_{J'2;J}\left[\frac{1}{2M}f(q^2)C^{nn';k}_{11;2}l_n q_{n'} + \frac{1}{(2M)^3}g(q^2)P\cdot l\sqrt{\frac{4\pi}{5}}\,Y_2^{k}(\mathbf{q})\right]$$

$$a = g_V \qquad c = \sqrt{3}g_A$$
$$b - a = \sqrt{3}g_M \qquad d = \sqrt{3}g_T$$
$$e = g_S \qquad h = \sqrt{3}g_P.$$

$$l^\mu\langle\beta|A_\mu|\alpha\rangle = C^{M'k;M}_{J'1;J}\varepsilon_{ijk}\varepsilon_{ij\lambda\eta}\frac{1}{4M}\left[c(q^2)l^\lambda P^\eta - d(q^2)l^\lambda q^\eta\right.$$
$$\left. + \frac{1}{(2M)^2}h(q^2)q^\lambda P^\eta q\cdot l\right]$$
$$+ C^{M'k;M}_{J'2;J}C^{nn';k}_{12;2}l_n\sqrt{\frac{4\pi}{5}}\,Y_2^{n'}(\mathbf{q})\frac{1}{(2M)^2}j_2(q^2)$$
$$+ C^{M'k;M}_{J'3;J}C^{nn';k}_{12;3}l_n\sqrt{\frac{4\pi}{5}}\,Y_2^{n'}(\mathbf{q})\frac{1}{(2M)^2}j_3(q^2),$$

for decay between isobaric analogs:

$$\langle I, I_z \pm 1|V_\mu{}^W|I, I_z\rangle,$$

$$a(0) = [(I \mp I_z)(I \pm I_z + 1)]^{1/2}$$
$$b(0) = a(0)\sqrt{\frac{J+1}{J}}(\mu_\beta - \mu_\alpha)$$
$$e(0) = f(0) = 0$$
$$g(0) = -a(0)\left(\frac{(J+1)(2J+3)}{J(2J-1)}\right)^{1/2}\frac{2M^2}{3}(Q_\beta - Q_\alpha),$$

where $x(q^2) = x_0 + x_1 q^2 \ldots$

Valence nucleon shell-model expressions for G-T, weak mag

This unpaired nucleon expression is incomplete for G-T transitions: (de-Shalit+Talmi Table 9.1)

THE VALUES OF $\langle\boldsymbol{\sigma}\rangle^2$ FOR SINGLE NUCLEON TRANSITIONS

j_f \ j_i	$l+\frac{1}{2}$	$l-\frac{1}{2}$
$l+\frac{1}{2}$	$\frac{2l+3}{2l+1}=\frac{j_f+1}{j_f}$	$4\,\frac{l+1}{2l+1}$
$l-\frac{1}{2}$	$\frac{4l}{2l+1}$	$\frac{2l-1}{2l+1}=\frac{j_f}{j_f+1}$

but it can lend qualitative understanding for why the G-T/Fermi ratio is so different in *n*, ^{19}Ne, ^{37}K... e.g. both μ and G-T transitions are smaller for $d_{3/2}$ proton because the orbital term partly cancels the intrinsic spin term.

Weak magnetism in G-T transitions (Wang+Hayes PRC 95 064313 (2017)):

$$\frac{d\omega}{dE_e} = \frac{G_F^2\cos^2\theta_C}{2\pi^3}\,p_e\,E_e(E_0-E_e)^2F(E_e,Z)g_A^2|\langle\vec{\Sigma}\rangle|^2 \times\left(1+\frac{4}{3}\left[\frac{\mu_v+\frac{\langle J_f||\vec{\Lambda}||J_i\rangle}{\langle J_f||\vec{\Sigma}||J_i\rangle}}{2\,M_N\,g_a}\right](2E_e-m_e^2/E_e-E_0)\right)$$

$$\delta_{LS}^{j_f j_i} = \frac{\langle nlj_f||\vec{\Lambda}||nlj_i\rangle}{\langle nlj_f||\vec{\Sigma}||nlj_i\rangle}$$

with $j_i=l\mp 1/2$ and $j_f=l\pm 1/2$

μ_v = 4.7 (isovector nucleon moment $\mu_p-\mu_n$)

$$\delta_{LS}^{--} = -(l+1),\quad \delta_{LS}^{-+} = -1/2,$$
$$\delta_{LS}^{+-} = -1/2,\quad \delta_{LS}^{++} = +l.$$

For reactor ν production, some simple estimates assumed the nucleon contribution $\pm$ 100%.

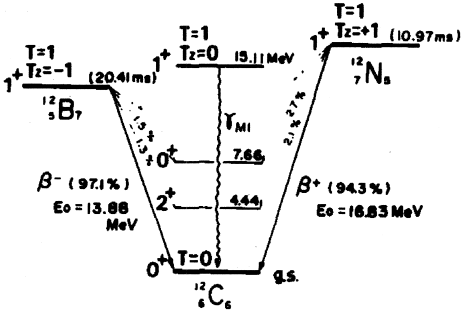
Weak Magnetism tests

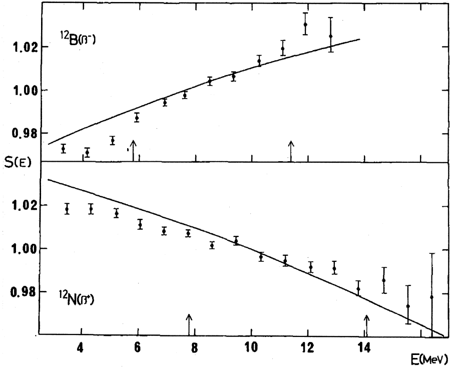
- For isobaric analog decays, the 'weak magnetism' $\frac{g_M}{2m}\sigma_{\mu\nu}k_\nu$ term is directly predicted by CVC by the 'anomalous' magnetic moment difference of the parent and daughter.
- For 'isospin mirror' Gamow-Teller decays, it is related to the isovector M1 γ-decay strength in the T_z=0 nucleus (Gell-Mann PhysRev 111 362 (1958)).

The k-dependence makes 20% distortions in the energy spectrum.

The axial vector $\frac{g_T}{2m}\sigma_{\mu\nu}k_\nu\gamma_5$ term, cancels in the difference unless there is a 2nd-class g_T.

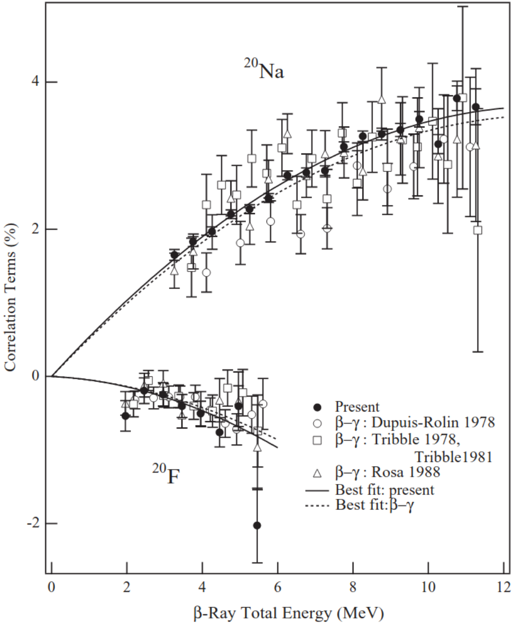
The results are consistent with the CVC prediction to $\sim$10% of the g_M term

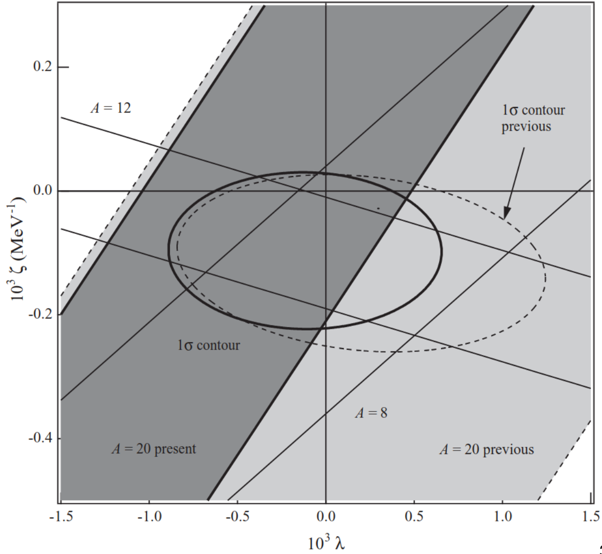




Further weak magnetism test:

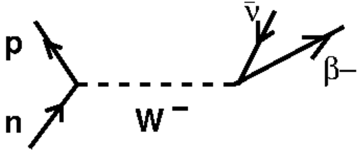
The angular distribution of β's is isotropic wrt alignment $\langle M_J^2 \rangle$ for Gamow-Teller decay. Results agree with CVC to $\sim$5%



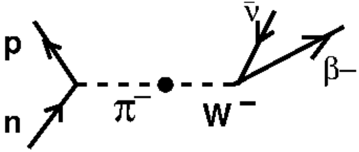


Minamisono PRC84 055501 (2011)   2nd class constraints →

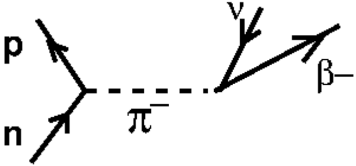
Sketch of lowest-order calculation of g_P Compare these diagrams:



A



B



C

Because W is short-range, C is same as B

For A (in a pure Gamow-Teller case), transition rate is:

$$T_{fi} = \frac{g}{2\sqrt{2}} V_{ud} \bar{u}_p \left(g_A \gamma_\mu \gamma_5 + i g_P k_\mu \gamma_5 \right) u_n \frac{1}{m_W^2} \frac{g}{2\sqrt{2}} \bar{u}_e \gamma_\mu (1+\gamma_5) v_{\nu e}$$

For C:

$$T_{fi} = g_{\pi NN} \sqrt{2} \bar{u}_p \gamma_5 u_n \frac{1}{k^2 + m_\pi^2} \frac{G}{\sqrt{2}} i f_\pi k_\mu \bar{u}_e \gamma_\mu (1+\gamma_5) v_{\nu e} V_{ud}$$

So C is like the g_P part of A; if we declare C responsible for all of it:

$$g_P(k^2) = \frac{g_{\pi NN}\sqrt{2} f_\pi}{k^2 + m_\pi^2}$$

In β decay this is small, but in μ capture it is a large contribution: (in computing the decay lifetime, g_P becomes multiplied by the lepton mass.)

Continuing g_P, weak and strong interactions together:

see Gorringe and Fearing Rev.Mod.Phys. 76 (2004) 1 for chiral perturbation theory
Further arguments (including PCAC below) give the 'Goldberger-Treiman' expression for the QCD-induced 'pseudoscalar' coupling:

$$g_P(q^2) = \frac{2m_\mu m_N}{m_\pi^2 - q^2} g_A(0)$$

This is now understood as the first term in an expansion using "chiral perturbation theory"; modern calculations $g_p(-0.88m_\mu^2) = 8.23$.
"Chiral perturbation theory" is a systematic expansion in $m_\pi / m_{\mathrm{nucleon}}$, guided by similar concepts to our considered "chiral EFT's", (small m_{quark}, π's as Goldstone bosons from the underlying broken chiral symmetry...) but a calculation, not with free parameters.
History: Experiments in radiative capture on hydrogen: 12.4$\pm$0.9$\pm$0.4
Experiments as of 2004 in ordinary μ capture on hydrogen: 10.5$\pm$1.8
As of 2004, not good enough to help yet: PSI was working on it
This rigorous prediction of low-energy QCD's effects on weak interaction was not working in 2004. A more accurate PSI experiment resolved the discrepancy with theory: Andreev Phys Rev Lett 110 012504 (2013) $g_p(-0.88\mu^2)$=8.06$\pm$0.55.

Conserved Vector Current and 'Partially Conserved Axial Current': qualitative

One consequence of the conserved 'V' vector current is that the equivalent g_V=1. I.e. the interaction between quarks goes directly over to the interaction between nucleons and nuclei because the 'vector current' is conserved.
People looked pretty hard to find a way to find an axial 'A' current that was also conserved (look at Feynman and Gell-Mann PR '57).

Wong writes Eq. 5-52:

$$\sum_{\mu=1}^{4} \frac{\partial V_\mu}{\partial x_\mu} = 0$$

and then by analogy

$$\sum_{\mu=1}^{4} \frac{\partial A_\mu}{\partial x_\mu} = \text{constant}\,\phi_\pi$$

where ϕ_π represents the pion field.

This predicts a relation between the weak coupling constants G_A and G_V, given by :

$$g_A \equiv \frac{G_A}{G_V} = \frac{f_\pi g_{\pi N}}{M_N c^2}$$

where f_π scales π decay and $g_{\pi N}$ can be deduced from π-nucleon scattering. This 'Goldberger-Treiman relation' predicts $|g_A| = 1.31$; experimental value is g_A=-1.259$\pm$0.004
This either 'confirms PCAC' or enforces that 'PCAC is a bad name for a poor approximation'.
Lattice QCD is at 1% accuracy for g_A
Note that this is all at momentum transfer $q^2\sim 0$: the constants are really 'form factors,' functions of momentum.

PCAC in more detail

Axial (hadronic) Current:

$$A_\mu = -i\frac{g}{2\sqrt{2}}\bar{u}(p_2)(g_A\gamma_\mu\gamma_5 + ig_P k_\mu\gamma_5)u(p_1)e^{i(p_1-p_2)\cdot x}$$

PCAC hypothesis: the non-conservation of this current is due entirely to pions, and A_μ becomes conserved as m_π goes to 0:

$$\partial_\mu A_\mu \overset{m_\pi\to 0}{\longrightarrow} 0$$

So evaluate the divergence of this current:

$$\partial_\mu A_\mu =$$

$$\frac{-ig}{2\sqrt{2}}\bar{u}(p_2)\left(g_A i\not{p}_1\gamma_5 - g_A i\not{p}_2\gamma_5 + g_P k^2\gamma_5\right)u(p_1)e^{i(p_1-p_2)\cdot x} =$$

using Dirac eq.

$$\frac{-ig}{2\sqrt{2}}\bar{u}(p_2)\left(2mg_A + g_P k^2)\gamma_5\right)u(p_1)e^{i(p_1-p_2)\cdot x}$$

By PCAC this vanishes as $m_\pi\to 0$, so:

$$g_A \overset{m_\pi\to 0}{\longrightarrow} \frac{g_P k^2}{2m} =$$

$$\frac{g_{\pi NN}\sqrt{2}f_\pi}{k^2+m_\pi^2}\frac{k^2}{2m} =$$

$$-\frac{g_{\pi NN}\sqrt{2}f_\pi}{2m}$$

the Goldberger-Treiman relation

Summary of hadronic weak current form factors in S.M.

- **Exact Predictions of CVC for vector current:**

1) g_V=1...: Experimental 0+$\rightarrow$ 0^+ *Ft* values same to $\approx$ 0.001. CKM unitarity has a 0.001 deficit at 2 to 3 σ. ($\pi^+ \rightarrow \pi_0 + \nu + \beta^+$ agrees to 0.005 (PIBETA)
2) g_M=3.70: Weak magnetism measured to $\approx$ 5% of its value
3) g_S=0: *Ft*, and relative helicity of leptons from β-ν correlation and $\pi \rightarrow e\nu$, show no evidence for scalar term at C_S<0.05 level.

- **Estimates from PCAC (Goldberger-Treiman) and similar:**

1) $g_A = \frac{g_{\pi NN}\sqrt{2}f_\pi}{2m} = -1.32$; Decay of neutron $\Rightarrow$ -1.26
2) $m_\mu g_P = \frac{g_{\pi NN}\sqrt{2}f_\pi m_\mu}{m_\pi^2} = 9.2$ Including chiral perturbation theory more like 8.0, PSI's μCAP experiment μ capture on hydrogen agrees well.
Charge Symmetry (G-parity): No 2nd-class currents: $f_3 \approx 0$, $g_2 \approx 0$
(The best tests of this SU(2) symmetry are still in β decay: similar tests in hadronic decays of τ)

- **V and A are the dominant known couplings for nuclear β decay. The most precise a_β measurement in the neutron diagrees badly, suggesting a large Lorentz tensor interaction. Interesting that a couple of simple surmises determined 6 couplings so well– reasonable to call it an "effective field theory" for the lepton-nucleon weak interaction. See Ando PhysLettB595 250 (2004) for an EFT of neutron β decay including radiative corrections.**

Clarification of g_V

I said g_V=1.00 was experimentally shown, which was pretty sloppy. Better to say g_V=1 is a prediction of CVC:

- **The $0^+ \to 0^+$ *Ft* values are experimentally constant, testing whether g_V is a constant for all transitions, but not necessarily g_V=1.000...**

- **Backing up, G_V is determined by**

$$\mathcal{F}t \equiv ft(1+\delta_R')(1+\delta_{NS}-\delta_C) = \frac{K}{2G_V^2\left(1+\Delta_R^V\right)}, \quad (1)$$

where $K/(\hbar c)^6 = 2\pi^3\hbar \ln 2/(m_e c^2)^5 = 8120.27648(26) \times 10^{-10}$ GeV^{-4}s, G_V is the vector coupling constant for semileptonic weak interactions, δ_C is the isospin-symmetry-breaking correction, and Δ_R^V is the transition-independent part of the radiative correction. The terms δ_R' and δ_{NS} comprise the transition-dependent part of the radiative correction, the

and then V_{ud} is determined by

$$V_{ud} = G_V/G_F, \quad (2)$$

where G_F is the well-known weak-interaction constant for muon decay. Once the value of V_{ud} is established it can be

So the present deficit in V_{ud} could also be a change for g_V from its value of 1 from electroweak unification.

E.g., *e* and μ weak couplings could be different.
Crivellin and Hoferichter PRL 125 111801 (2020) consider keeping CKM unitarity while considering constraints from $(\pi \to e\nu)/(\pi \to \mu\nu)$

Axial vector is not conserved. Is g_A the same in nuclei? G-T ("Ikeda") sum rule:

define the sum rule strength

$$S_\pm = G_A^{-2} \sum_f |\langle f | O_{\mathrm{GT}}(\beta^\pm) | i \rangle|^2$$

$$\begin{aligned} S_\pm &= G_A^{-2} \sum_f \langle f | O_{\mathrm{GT}}(\beta^\pm) | i \rangle^* \langle f | O_{\mathrm{GT}}(\beta^\pm) | i \rangle \\ &= G_A^{-2} \sum_f \langle i | O^\dagger_{\mathrm{GT}}(\beta^\pm) | f \rangle \langle f | O_{\mathrm{GT}}(\beta^\pm) | i \rangle \\ &= G_A^{-2} \langle i | O^\dagger_{\mathrm{GT}}(\beta^\pm) O_{\mathrm{GT}}(\beta^\pm) | i \rangle \end{aligned}$$

operators involved here have the following p

$$\sigma^\dagger_\mu = (-1)^\mu \sigma_{-\mu} \qquad \tau^\dagger_\mp = \tau_\pm$$

$$\begin{aligned} S_+ &= \langle i | \sum_{k=1}^{A} \sum_\mu (-1)^\mu \sigma_{-\mu}(k) \tau_+(k) \sigma_\mu(k) \tau_-(k) | i \rangle \\ &= \langle i | \sum_{k=1}^{A} \sigma^2(k) \tau_+(k) \tau_-(k) | i \rangle \end{aligned}$$

In a spherical basis, the scalar product m

$$\boldsymbol{J} \cdot \boldsymbol{V} = \sum_q (-1)^q J_{1q} V_{1,-q}$$

$$\tau_+\tau_- | p \rangle = | p \rangle \qquad \tau_+\tau_- | n \rangle = 0$$

expectation value of σ^2 is 3.

$$S_+ = \langle i | \sum_{k=1}^{Z} \sigma^2(k) | i \rangle = 3Z$$

Similarly, S_-= 3N , and $S_+ - S_-$=3(Z-N)
Detailed studies with high-Q β decay (and (p,n) and (n,p) reactions at 100-200 MeV)found ≈ 75% of the sum rule.
Recent calculations (Gysberg Nat Phys 15 428 2019) reproduce GT strength with about 5-10% accuracy, combining chiral EFT's with accurate many-body techniques and considering '2-body currents'

Jackson, Treiman, Wyld 1957 wrote down 4-Fermi vertex interaction H for nucleon beta decay. (These could be written in a more natural helicity basis...)

You construct Lorentz-invariant quantities, i.e. a Lorentz scalar, from the possible objects which Lorentz transform like vectors, axial vectors, scalars, tensors, pseudoscalars (it turns out all combinations of more Dirac matrices reduce to these). Assuming pointlike high-mass-bosons only, one would now call this an EFT:

$$\begin{aligned} H_{\mathrm{int}} = & (\bar{\psi}_p \psi_n)(C_S \bar{\psi}_e \psi_\nu + C_S' \bar{\psi}_e \gamma_5 \psi_\nu) \\ & + (\bar{\psi}_p \gamma_\mu \psi_n)(C_V \bar{\psi}_e \gamma_\mu \psi_\nu + C_V' \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu) \\ & + \tfrac{1}{2}(\bar{\psi}_p \sigma_{\lambda\mu} \psi_n)(C_T \bar{\psi}_e \sigma_{\lambda\mu} \psi_\nu + C_T' \bar{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu) \\ & - (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n)(C_A \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu + C_A' \bar{\psi}_e \gamma_\mu \psi_\nu) \\ & + (\bar{\psi}_p \gamma_5 \psi_n)(C_P \bar{\psi}_e \gamma_5 \psi_\nu + C_P' \bar{\psi}_e \psi_\nu) \\ & + \text{Hermitian co} \end{aligned}$$

Quark-lepton interactions have been found experimentally to be V,A only so far. V is assumed conserved (like electric charge), so C_V=1 is often assumed. QCD still can change A, and ‘induce’ all the other terms for hadron-lepton interactions, changing all these constants but C_V. We’ve seen how this creates interesting ways to test QCD’s influence on weak interactions, and we’ve already seen $|C_A|$ = 1.26...

I.e. this looks a lot like the S.M. quark-lepton Lagrangian but of course we have to be careful about the C_x’s

Jackson, Treiman, Wyld 1957 wrote down observables before angular integration, and the answers

$$\omega(\langle J\rangle | E_e,\Omega_e,\Omega_\nu)dE_e d\Omega_e d\Omega_\nu$$
$$= \frac{1}{(2\pi)^5} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \xi \Big\{ 1 + a\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e}$$
$$+ c\left[\frac{1}{3}\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu} - \frac{(\mathbf{p}_e\cdot\mathbf{j})(\mathbf{p}_\nu\cdot\mathbf{j})}{E_e E_\nu}\right]\left[\frac{J(J+1)-3\langle(\mathbf{J}\cdot\mathbf{j})^2\rangle}{J(2J-1)}\right]$$
$$+ \frac{\langle\mathbf{J}\rangle}{J}\cdot\left[A\frac{\mathbf{p}_e}{E_e} + B\frac{\mathbf{p}_\nu}{E_\nu} + D\frac{\mathbf{p}_e\times\mathbf{p}_\nu}{E_e E_\nu}\right]\Big\}.$$

Rather than use JTW's answers here,

we'll assume the functional form of the correlations. In limiting cases, assumptions about S.M. lepton helicity will then let us deduce the S.M. predictions soon.
The S.M. weak interaction makes left-handed leptons and right-handed antileptons in decays, Helicity $\hat{s}\cdot\hat{p}$
Note $\frac{p}{E}$ is, of course, $\frac{v}{c}$. One can always boost to a frame moving faster than a massive particle– reversing $\hat{p}$ but preserving $\hat{s}$. That's intuitively why there's a factor of $\frac{v}{c}$ multiplying the helicities.

Re: the 'Fierz interference' term $b\frac{m_e}{E_e}$,
product of a SM term with normal helicity and a ~~SM~~ term with non-normal helicity:
$\sqrt{1+\frac{p_e}{E_e}} \times \sqrt{1-\frac{p_e}{E_e}} = \sqrt{1-\frac{p_e^2}{E_e^2}} = \frac{m_e}{E_e}$ **take care with particle physics 'chirality' vs. 'helicity'**

Reference for non-Dirac treatment: R. Hong, M. Sternberg, A. Garcia, "Helicity and nuclear β decay correlations," American Journal of Physics 85 p 45 (2017).

Measure ν helicity $\epsilon = \hat{s}_\nu \cdot \hat{k}_\nu$ directly: transfer $\hat{s}_\nu$ to γ circular polarization; boost $\vec{k}_\gamma$ by $\pm\vec{k}_\nu$

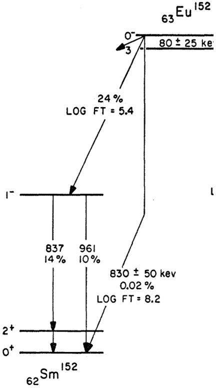
Goldhaber, Grodzins, Sunyar
Phys Rev 109 1015 (Dec 1957)

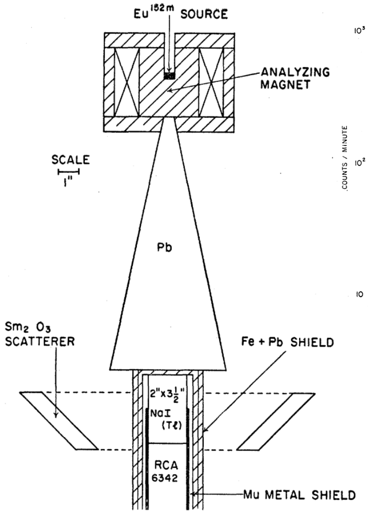
$$e^- + {}^{152m}\mathrm{Eu} \rightarrow \nu + {}^{152}\mathrm{Sm}$$

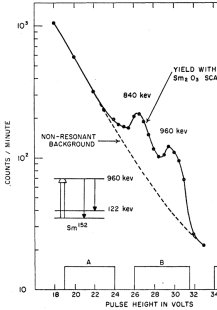
- ν with $\hat{s}$ = -1 populates $\langle J_z \rangle = 0, +1$ not -1
- So γ is circularly polarized– transmission through magnet depends on iron polarization: $\frac{N_+ - N_-}{N_+ + N_-} = 0.017 \pm 0.003$
- Upward ν boosts γ momentum so it can be absorbed on-resonance

$\Rightarrow$ ν helicity -1 $\pm$ 10%

($\bullet$ $\bar{\nu}$ helicity $\sim$ +1
Palathingal PRL 524 24 '69)







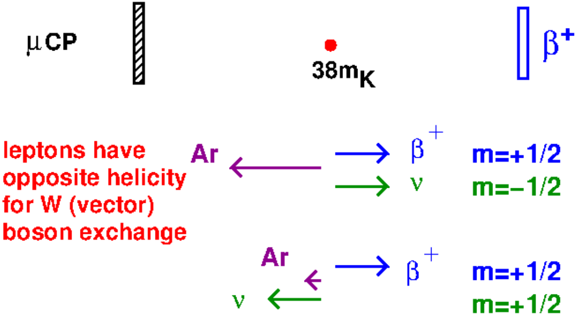
Surprisingly enough, this is the best direct measurement of ν helicity = $\hat{s}_\nu \cdot \hat{k}_\nu$

The β-ν angular distribution in the SM

$$W[\theta_{\beta\nu}] = 1 + a\,\frac{v_\beta}{c}\cos\theta_{\beta\nu}$$

For ^{38m}K, $0^+ \rightarrow 0^+$ decay:

***a*= +1 'Proof':**



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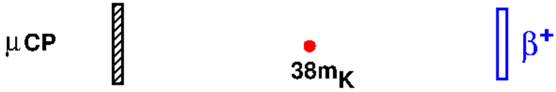
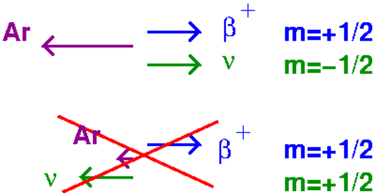


leptons have opposite helicity for W (vector) boson exchange



For scalar exchange, lepton helicities are same: a= -1

No nuclear structure corrections until 10^{-6} accuracy

Note $a_{\beta\nu}$ depends on the relative helicity of β and ν, but not the absolute sign. The observable is parity-even, and is not actually sensitive to parity violation.

β-ν correlation from recoil momentum spectrum Kofoed-Hansen Dan. Mat. Fys. Medd. 28 nr9 (1954)

The recoil momentum spectrum is straightforward and analytic:
If we write angular distribution in terms of *E* (β total energy), θ (β-ν angle), p (β momentum), q (ν momentum) (it's understood we have to evaluate q to conserve energy-momentum; it's not a free parameter)

$$P(E,\theta)dEd\Omega_\theta =$$

$$F(Z,E)pEq^2\left(1+\frac{b}{E}+a\frac{p}{E}\cos\theta\right)dEd\Omega_\theta$$

Then if the recoil momentum is *r*, energy conservation E+q=E_0 (E_0=Q+m_β), then we just use law of cosines:

$$p^2+q^2+2pq\cos\theta=r^2$$

differentiate θ with respect to *r*:

$$|\sin\theta d\theta| = 2d\Omega_\theta = \frac{r}{pq}dr$$

we immediately get the recoil momentum spectrum

$$P(E,r) =$$

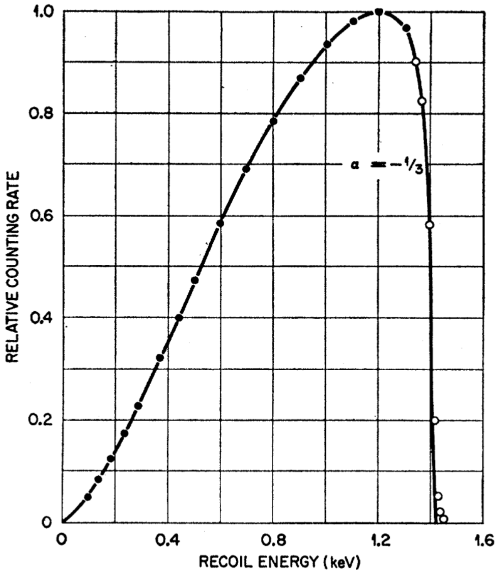
$$\frac{1}{2}F(Z,E)\left(rEq+brq+r\frac{a}{2}(r^2-p^2-q^2)\right)dEdr$$

at fixed *E*, it's linear in recoil energy *R*

$$P(E,R)dEdR =$$

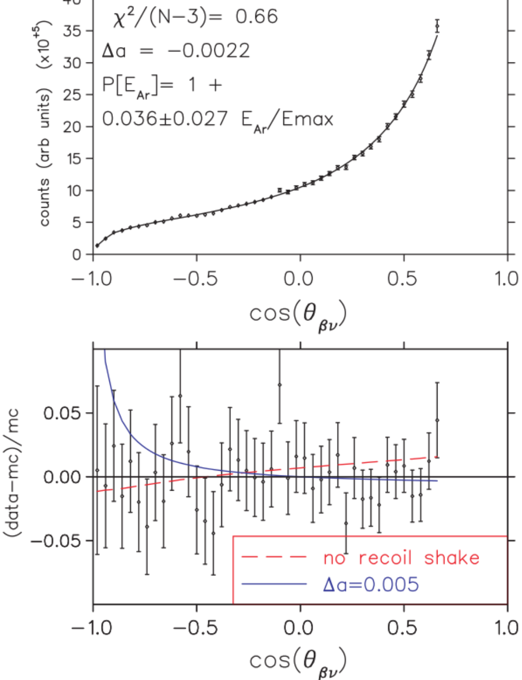
$$\frac{M}{2}F(Z,E)\left(Eq+bq+\frac{a}{2}(2MR-p^2-q^2)dEdR\right)$$

1963 ^{6}He $a_{\beta\nu}$ definitive evidence for V,A instead of S,T.



Feynman&Gell-Mann paper PR 109 193 (1957) proposing both CVC and V$\pm$A: "These theoretical arguments seem to the authors to be strong enough to suggest that the disagreement with the He^6 recoil experiment... indicates that these experiments are wrong."

Then Johnson, Pleasonton, Carlson PhysRev 132 1149 (1963)
$a_{\beta\nu}$ = -0.3308 $\pm$ 0.0030
agreed much better with V,A (-1/3) than S,T (+1/3).
(This $0^+ \rightarrow 1^+$ decay is pure Gamow-Teller, hence axial vector, sensitive to Lorentz axial vector and tensor interactions, though not the sign wrt vector and scalar)



$\beta - \nu$ correlation for pure Fermi transition $^{38m}K\ 0^+ \to 0^+$

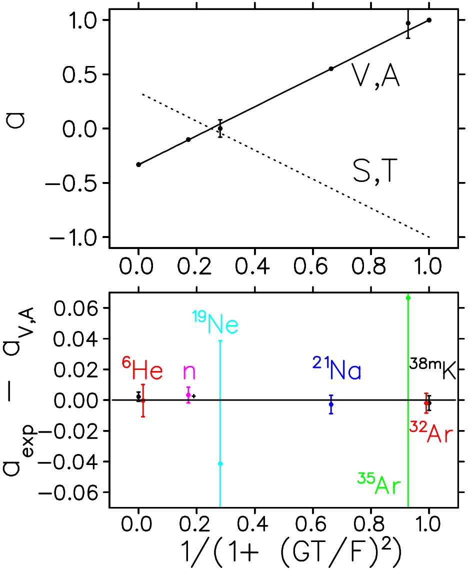
Angular distribution of ν wrt β determined from other observables (except E_β).

Gorelov PRL 94 142501 (2005)
$\tilde{a}$=0.9981 $\pm$ 0.0030(stat) $\pm$ 0.0037(syst)

(Adelberger ^{32}Ar also $0^+ \to 0^+$
PRL 83 1299 (1999) (err. 83 3101)
$\tilde{a}$=0.9989 $\pm$ 0.0052(stat) $\pm$ 0.0039(syst)

Together constrain that Lorentz scalar contribution is small $\to$

Summarizing info on Lorentz structure from β-ν correlation



Interaction is mostly vector and axial vector, i.e. V and A

[Except aSPECT has difference in *a* for neutron $(2.57 \pm 0.84)x10^{-3}$ Explainable by a finite Lorentz tensor allowed by other nuclear β decay]

For the sign between them, we need to consider parity violation →

Symmetries: Continuous, Discrete

- Noether's theorem (1915):

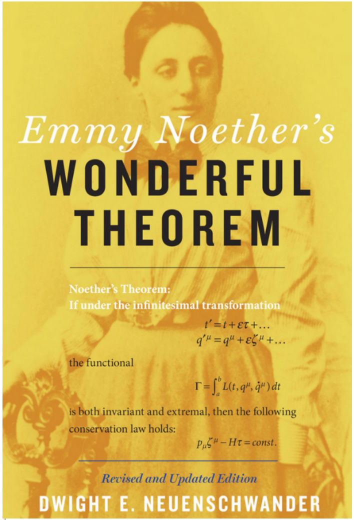
Continuous symmetry	$\rightarrow$	Conserved quantity
Time-translational invariance	$\rightarrow$	Energy
Space-translational invariance	$\rightarrow$	Momentum
Rotational invariance	$\rightarrow$	Angular momentum
(Laplace-Runge-Lenz vector)	$\rightarrow$	name?

THE LATE EMMY NOETHER.

Professor Einstein Writes in Apprecia-tion of a Fellow-Mathematician.

To the Editor of The New York Times:

gan. In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance in the development of the present-day younger generation of mathematicians. Pure mathematics is, in its way, the poetry of logical ideas. One seeks the most general ideas of operation which will bring together in simple, logical and unified form the largest possible circle of formal relationships. In this effort toward logical beauty spiritual formulae are discovered necessary for the deeper penetration into the laws of nature.



In Ted Chiang's "Story of Your Life" [Movie "Arrival"]: aliens think in terms of the action, not position and momentum

- Discrete symmetries in quantum mechanics: Parity, Time reversal $\rightarrow$

Historical Ideas about *P*, *T* breaking

- Wigner considered implications of P, T symmetry conservation in atomic spectra 1926-28. Showed $\langle T\psi_i, T\psi_f\rangle = \langle \psi_f, \psi_i\rangle^*$

"In quantum theory, invariance principles permit even further reaching conclusions than in classical mechanics." (D. Gross, Physics Today 48 46 (1995))

- Weyl 1931 considered *C*, *P*, *T* and *CPT* in "Maxwell-Dirac theory": C $\Rightarrow$ Dirac eq. negative energy states had to have same mass as the e^- plato.stanford.edu
- From "CP Violation Without Strangeness" Khriplovich and Lamoreaux:

1949 Dirac "I do not believe there is any need for physical laws to be invariant under reflections in space and time although the exact laws of nature so far known do have this invariance."

- 1956 Lee and Yang proposed $\not{P}$ in weak decays to fix the θ-τ puzzle
- Feynman gives Ramsey 50:1 odds $\not{P}$ would not be observable
  Ramsey experiment starting at ORNL gets derailed by fission experiments...
  it's OK, Ramsey won 1989 Nobel for his fringes
- 1957 3 simultaneous experimental measurements of $\not{P}$ $\rightarrow$

Parity (From A. Zee "Fearful Symmetry")

As of 1956, we thought all interactions respected parity

Parity operator

$P\,\psi(\vec{r}) \rightarrow \pm\,\psi(-\vec{r})$

P  $=$ 

1957:

$\tau - \theta$ Puzzle

+ μ decay

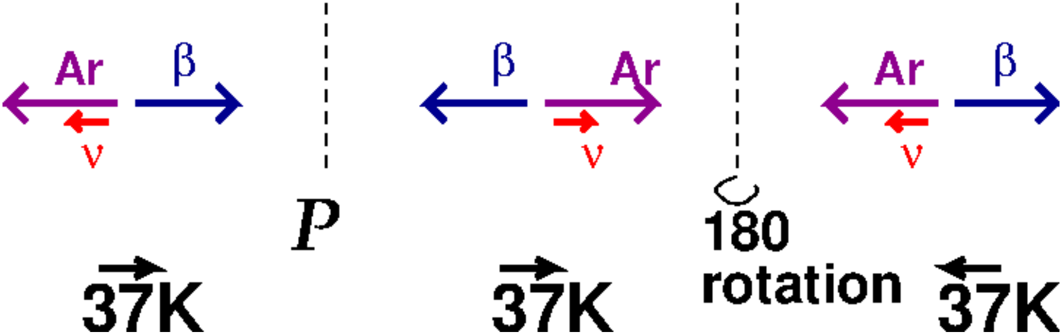
+ ^{60}Co decay $\Rightarrow$

P  $\neq$ 

Decays: Parity Operation can be simulated by Spin Flip

Under Parity operation *P*:

$\vec{\mathbf{r}} \rightarrow -\vec{\mathbf{r}}$ $\quad\quad$ $\vec{\mathbf{p}} \sim \frac{d\vec{\mathbf{r}}}{dt} \rightarrow -\vec{\mathbf{p}}$ $\quad\quad$ $\vec{\mathbf{J}}=\vec{\mathbf{r}} \times \vec{\mathbf{p}} \rightarrow +\vec{\mathbf{J}}$

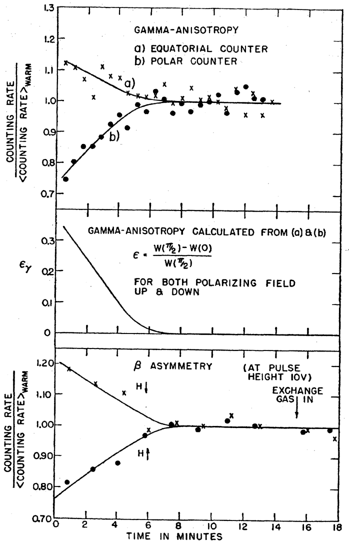


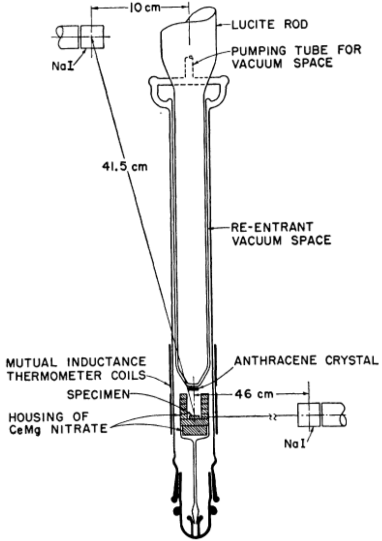
$\Rightarrow$ **A spin flip corresponds exactly to *P* reversal**

Decays don't exactly test *T*-reversal symmetry

One experimental discovery of parity violation

Wu, Ambler, Hayward, Hopper, Hobson, PR 105 1413 Feb ’57





Dilution Refrigerator to spin-polarize with nuclear polarization $P = \langle \frac{J_z}{J} \rangle$

$^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni} + \beta^- + \bar{\nu}$

$$W[\theta] = 1 + PA\hat{\mathbf{J}} \cdot \frac{\vec{p}_\beta}{E_\beta}$$

$$= 1 + AP\frac{v}{c}\cos[\theta]$$

$$A_{\beta-} \approx -1.0$$

Note: $5^+ \to 4^+$ Null if left-handed $\beta^- \to \vec{J}$

m_J^i=+5

m_β=-1/2, m_J^f=+4

Lookup for allowed β-decay correlations

Jackson Treiman Wyld NuclPhysA 4 206 (1957)

± 1, no nuclear parity change, $J = 0 \to J' = 0$ forbidden. J and J' are the angular momenta of the original and final nuclei, $\delta_{J'J}$ is the Kronecker delta symbol and

$$\lambda_{J'J} = \begin{cases} 1 & J \to J' = J-1 \\ \dfrac{1}{J+1} & J \to J' = J \\ -\dfrac{J}{J+1} & J \to J' = J+1 \end{cases} \quad (A.1)$$

$$\Lambda_{J'J} = \begin{cases} 1 & J \to J' = J-1 \\ -\dfrac{(2J-1)}{J+1} & J \to J' = J \\ \dfrac{J(2J-1)}{(J+1)(2J+3)} & J \to J' = J+1. \end{cases} \quad (A.2)$$

Z is the atomic number of the final nucleus, α is the fine structure constant, and $\gamma = (1-\alpha^2 Z^2)^{\frac{1}{2}}$.

For pure G-T: $A_{\beta\pm} = \pm\, \lambda_{J',J}$

Textbooks with calculations:
***a*, the 'β — ν correlation':**
Halzen&Martin "Quarks&Leptons,"
my notes ph505jbVIII_2005_aBetaNu_WithDirac.pdf

Melconian's notes include Fierz term!
***A*, the 'β asymmetry wrt spin':**
Greiner and Müller "Gauge Theory of Weak Interactions"
Towner's notes within mine

upper sign for β^-, lower sign for β^+

$$\xi = |M_F|^2(|C_S|^2+|C_V|^2+|C'_S|^2+|C'_V|^2) + |M_{GT}|^2(|C_T|^2+|C_A|^2+|C'_T|^2+|C'_A|^2) \quad (A.3)$$

$$a\xi = |M_F|^2 \left\{[-|C_S|^2+|C_V|^2-|C'_S|^2+|C_V|^2] \mp \frac{\alpha Z m}{p_e} 2\ \mathrm{Im}\ (C_S C_V{}^* + C'_S C'_V{}^*)\right\} + \frac{|M_{GT}|^2}{3}\left\{[|C_T|^2-|C_A|^2+|C'_T|^2-|C'_A|^2] \pm \frac{\alpha Z m}{p_e} 2\,\mathrm{Im}\,(C_T C_A{}^* + C'_T C'_A{}^*)\right\} \quad (A.4)$$

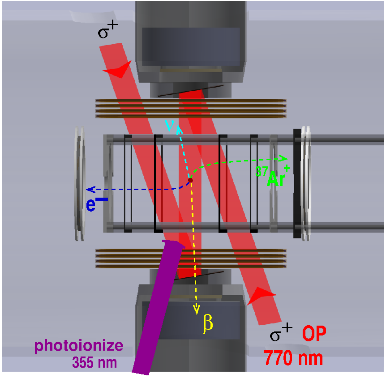
$$b\xi = \pm 2\gamma\ \mathrm{Re}[|M_F|^2(C_S C_V{}^* + C'_S C'_V{}^*) + |M_{GT}|^2(C_T C_A{}^* + C'_T C'_A{}^*)] \quad (A.5)$$

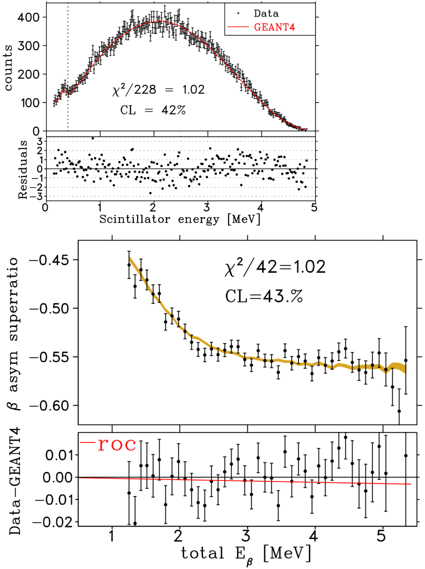
$$c\xi = |M_{GT}|^2 \Lambda_{J'J}\left[|C_T|^2-|C_A|^2+|C'_T|^2-|C'_A|^2 \pm \frac{\alpha Z m}{p_e} 2\,\mathrm{Im}\,(C_T C_A{}^* + C'_T C'_A{}^*)\right] \quad (A.6)$$

$$A\xi = |M_{GT}|^2 \lambda_{J'J}\left[\pm 2\ \mathrm{Re}(C_T C'_T{}^* - C_A C'_A{}^*) + \frac{\alpha Z m}{p_e} 2\ \mathrm{Im}\ (C_T C'_A{}^* + C'_T C_A{}^*)\right] + \delta_{J'J} M_F M_{GT} \sqrt{\frac{J}{J+1}}\left[2\ \mathrm{Re}\ (C_S C'_T{}^* + C'_S C_T{}^* - C_V C'_A{}^* - C'_V C_A{}^*) \pm \frac{\alpha Z m}{p_e} 2\ \mathrm{Im}\ (C_S C'_A{}^* + C'_S C_A{}^* - C_V C'_T{}^* - C'_V C_T{}^*)\right] \quad (A.7)$$

+ B_ν, ~~T~~ D, ...

β^+ asymmetry ^{37}K data





Fenker et al. Phys Rev Lett 120, 062502 (2018)

A_β[experiment]= -0.5707 $\pm$ 0.0019

A_β[theory] = -0.5706 $\pm$ 0.0007

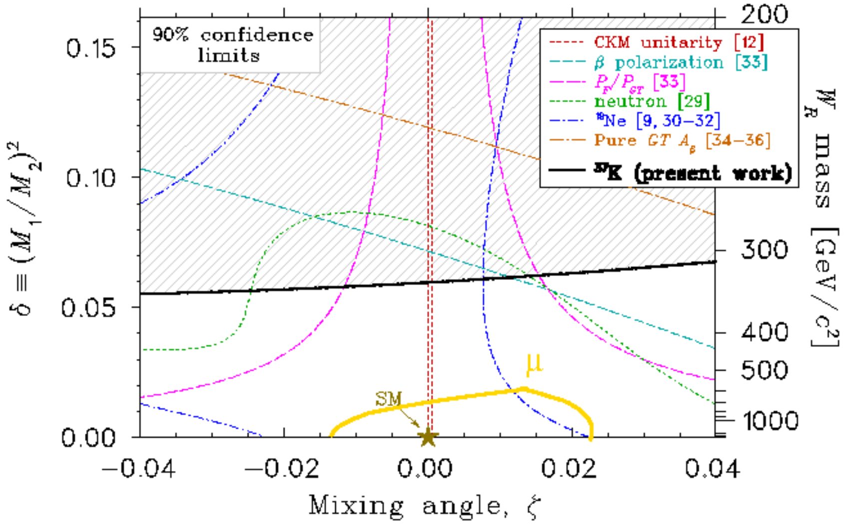
theory prediction needs GT/F ratio from $t_{1/2}$

The best fractional accuracy achieved in nuclear or neutron β decay

TEXAS A&M UNIVERSITY

Still no wrong-handed ν's

TRIUMF

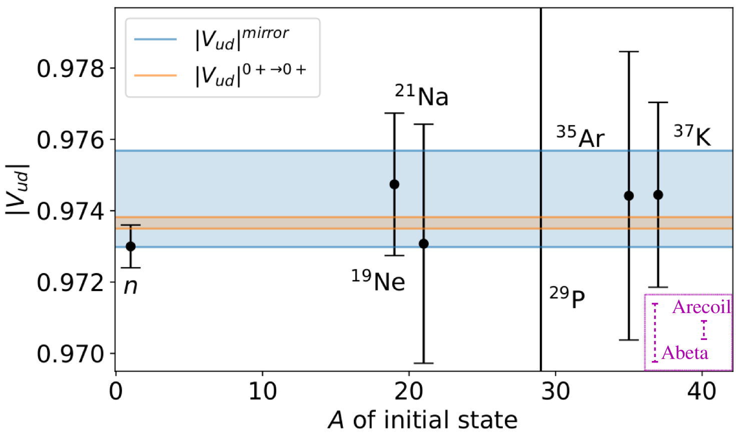


Extra W' with heavier mass, couples to wrong-handed ν_R

We can evade TWIST limits by assuming the muon ν_R is heavy

LHC $M'_W > 3.7$ TeV 90%

Weak interaction: same strength, all nuclei?



Hayen and Severijns, arXiv:1906.09870 (June 2019)

Deduced V_{ud} from mirror decays

Are people overestimating their uncertainties? We aren't ☺

We project to reach 0.0005 accuracy, as good as any $0^+ \to 0^+$ except ^{26m}Al.

Assumes 5% isospin breaking calculation.

TRIUMF

Physics and time reversal

When t $\rightarrow$ -t, does anything change?

- **Wave Equ. is 2nd-order in t: $\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ symmetric in t**
- **Heat Equ. is 1st-order in t: $\nabla^2 u = -\frac{\partial u}{\partial t}$ t$\rightarrow$ -t, boom?**

'Dissipation', like friction... The arrow of time remains a research problem in stat mech, but it's not from (known) particle physics

- **Schroedinger Equation is 1st order: $i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$**

'Take the complex conjugate'

(but see Dressel et al. PRL 119 220507 (2017)
"Arrow of Time for Continuous Quantum Measurements")

Microscopic physics was thought to be symmetric in t

Simulating $\not T$ in decays?

We've constructed an angular correlation, a scalar observable, by a dot product of two vectors

$$1 + \hat{p} \cdot \hat{J}$$

which is odd under *P* as we need

($\vec{p}$ is even, $\vec{J} = \vec{r} \times \vec{p}$ is odd)

But $\vec{J}$ is odd under *T*, not even

So we need at least 3 vectors to have a T-odd scalar observable,
the scalar triple product $\vec{v1} \cdot (\vec{v2} \times \vec{v3})$

An example $\rightarrow$

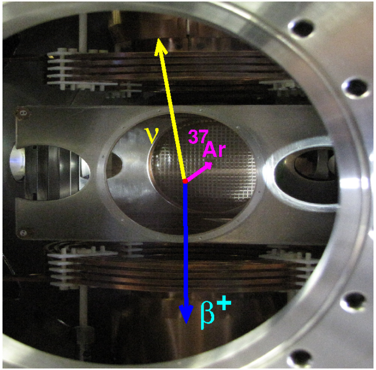
TRIUMF $\not T$ correlation of 3 of 4 momenta

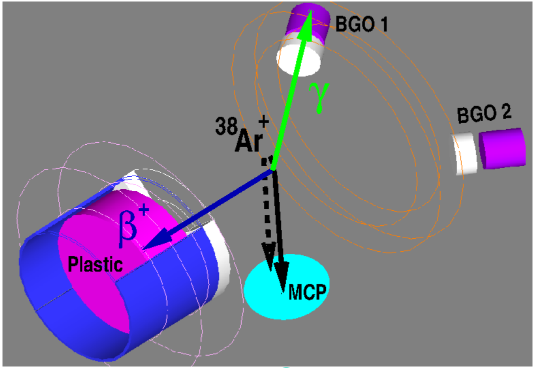
$t \to -t \Rightarrow \vec{p} \propto \frac{d\vec{r}}{dt} \to -\vec{p}$

but $\vec{p}_{recoil} \cdot \vec{p}_\beta \times \vec{p}_\nu \equiv 0$ ☹

$\vec{p}_\nu \cdot \vec{p}_\beta \times \vec{p}_\gamma = -\vec{p}_{recoil} \cdot \vec{p}_\beta \times \vec{p}_\gamma$

$\xrightarrow{t \to -t} \vec{p}_{recoil} \cdot \vec{p}_\beta \times \vec{p}_\gamma$

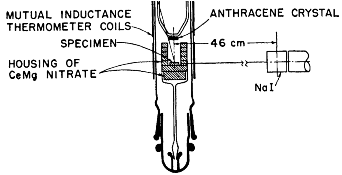


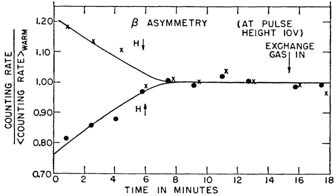


- We can test symmetry of apparatus with coincident pairs ☺
- Not exact. Outgoing particles interact $\to$ fake $\not T$

TRIUMF

Parity broken, why not $\not{T}$ime?





Immediately after $\not{P}$arity was seen to be totally broken in β decay ('ν left-handed')

Wu, Ambler, Hayward, Hopper, Hobson, PR 105 (1957) 1413

Many T-odd observables were proposed:

PHYSICAL REVIEW VOLUME 106, NUMBER 3

Possible Tests of Time Reversal Invariance in Beta Decay

J. D. Jackson,* S. B. Treiman, and H. W. Wyld, Jr.
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received January 28, 1957)

Need scalar triple products of 3 vectors: observables involving spin

$$D\hat{J}\cdot\frac{\vec{p}_\beta}{E_\beta}\times\frac{\vec{p}_\nu}{E_\beta} \qquad R\vec{\sigma}_\beta\cdot\hat{\mathbf{J}}\times\frac{\vec{\mathbf{p}}_\beta}{E_\beta}$$

are consistent with $\not{T} < 0.001$

but some has been found $\rightarrow$

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$\not{CP}$ discovered in $K\bar{K}$ meson decays in 1963, though not much (Cronin and Fitch Nobel prize 1980)

Quark eigenstates in the weak interaction:

To explain some weak decays we saw,

$|u\rangle \rightarrow |d\rangle + \epsilon|s\rangle$ **i.e.** $|u\rangle \rightarrow \cos(\theta_C)|d\rangle + \sin(\theta_C)|s\rangle$

Maybe one reason for 3 families of particles,

$\rightarrow$ 3x3 unitary "CKM" matrix between $|d\rangle, |s\rangle, |b\rangle$

There is one complex phase, which leads to this type of $\not{CP}$
A reason for 3 generations of particles?

That one phase is consistent with $\not{CP}$ in $K\bar{K}$ and $B\bar{B}$ systems

There have been hints in $K\bar{K}$ and $B\bar{B}$ of more $\not{CP}$ than in the standard model,

$p\bar{p} \to \mu^+\mu^+$ or $\mu^-\mu^-$ $\not{CP}$ at 3.6 σ Abazov PRD 2014 Fermilab;
so this 2001 cartoon was a little premature $\to$

J. Fabergé. CERN Courier. 6, No. 10. 193 (October 1966). [Courtesy of Madame Fabergé.]

T2K ν_μ oscillations different from $\bar{\nu}_\mu$ at 2 to 3 σ Nature 580 339 (2020)

$\not{CP}$ could have some utility for cosmology $\to$

The excess of matter over antimatter can come from $\not{CP}$

Sakharov JETP Lett 5 24 (1967) used $\not{CP}$ to generate the universe's excess of matter over antimatter:

- $\not{CP}$,
- **baryon nonconservation, and**
- **nonequilibrium.**

But known $\not{CP}$ is too small by 10^{10}, so 'we' need more to exist. **Caveats:**

- **You could use $\not{CPT}$ [Dolgov Phys Rep 222 (1992) 309]**
- **We need $\not{CP}$ in the early universe, not necessarily now**

So we look for more $\not{CP}$. How is this related to $\not{T}$?

this seems a little abstract
concrete demonstrative example from Ramsey-Musolf at INT 2020
$\not{CP}$ explaining T2K's ν vs. $\bar{\nu}$ result lets heavy *N* decay this way in some models

www.int.washington.edu/talks/WorkShops/int_20_2b/People/Ramsey-Musolf_M/Ramsey-Musolf.pdf

Neutrinos and the Origin of Matter

- *Heavy neutrinos decay out of equilibrium in early universe*
- *Majorana neutrinos can decay to particles and antiparticles*
- *Rates can be slightly different (CP violation)*

$$\Gamma(N \to \ell H) \neq \Gamma(N \to \bar{\ell} H^*)$$

- *Resulting excess of leptons over anti-leptons partially converted into excess of quarks over anti-quarks by Standard Model sphalerons*

$\not{T}$ is related to $\not{CP}$ by the "CPT Theorem"

"All local Lorentz invariant QFT's are invariant under CPT"
Schwinger Phys Rev 82 914 (1951)
Lüders, Pauli, Bell 1954

- Gravity $\rightarrow$ not flat:

K meson experiments Adler PhysLettB 364 (1995) 239 test $\not{CPT}$ to within 1000x expected from quantum gravity

- Strings not 'local'

Proofs still pursued $\rightarrow$

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On the CPT theorem

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ABSTRACT

We provide a careful development and rigorous proof of the CPT theorem within the framework of mainstream (Lagrangian) quantum field theory. This is in contrast to the usual rigorous proofs in purely axiomatic frameworks, and non-rigorous proof-sketches in the mainstream approach. We construct the CPT transformation for a general field directly, without appealing to the enumerative classification of representations, and in a manner that is clearly related to the requirements of our proof. Our approach applies equally in Minkowski spacetimes of any dimension at least three, and is in principle neutral between classical and quantum field theories: the quantum CPT theorem has a natural classical analogue. The key mathematical tool is that of complexification; this tool is central to the existing axiomatic proofs, but plays no overt role in the usual mainstream approaches to CPT.

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Assuming CPT, $\not{CP} \Leftrightarrow \not{T}$ in most physics theories
The matter excess then motivates $\not{T}$ searches

EDM in a fundamental particle breaks T: this is exact

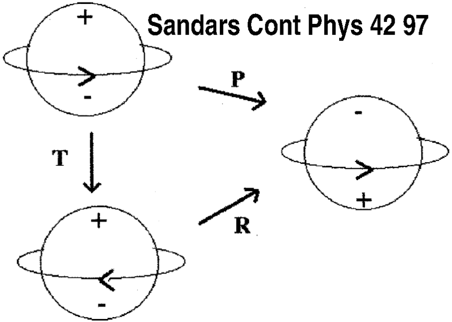
Landau, Nucl. Phys. 3 (1957) p. 127

Electric Dipole moment $\vec{d} = \sum q_i \vec{r}_i$

Since the angular momentum is the only vector in the problem, $\vec{d} = a\vec{J}$

Under T, $\vec{J} \overset{t\to -t}{\longrightarrow} -\vec{J} \quad \vec{d} \overset{t\to -t}{\longrightarrow} +\vec{d}$

If the physics is invariant under T, this is a contradiction, $\Rightarrow a = 0$



[• The other logical possibility: there are 2 states, with opposite sign of the EDM, and T just formally changes one state to the other.
For most fundamental particles, we know there aren't 2 states
Why do we know the electron doesn't have 2 states?
E.g. some polar molecules have a dipole moment listed in tables, which produces degenerate states and does not break T ...]

Schiff's Theorem: does a nuclear EDM make an atomic EDM?

Schiff's Theorem PR 132 2194 (1963): **The nuclear electric dipole moment $d_{\text{nuclear}} = \sum q_i r_i \hat{r}_i$ causes the atomic e^-'s to rearrange themselves so they develop an opposite dipole moment. In the limit of nonrelativistic e^-'s and a point nucleus, the e^-'s dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes.**

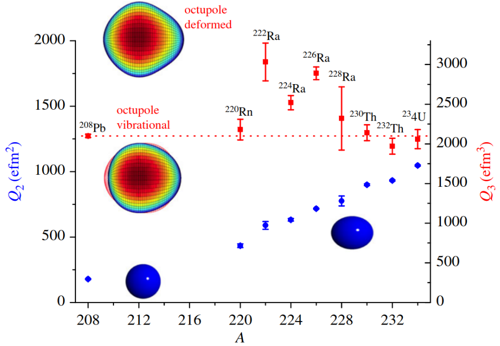
(For the e^-'s EDM, there is 'antiscreening,' and $d_{\text{atom}} \overset{Z\gg1}{\ggg} d_{e-}$ Sandars Phys Lett 14 194 (1965)**)**

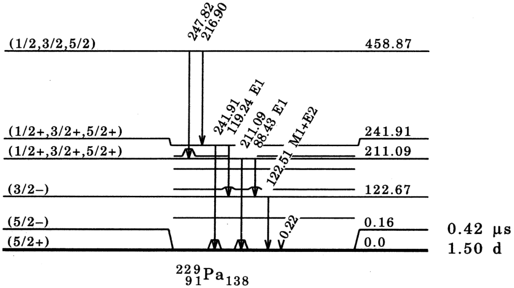
The Schiff moment S involves $\sum q_i r_i^2 \hat{r}_i$ does not get screened completely:
$\langle S\rangle = \sum q_i(r_i^2 - \frac{5}{3}\langle R_{\text{ch}}^2\rangle) \approx R_{\text{nucleus}}^2 d_{\text{nucleus}}$, **so** $d_{\text{atom}}/d_{\text{nucleus}} \sim R_{\text{nucleus}}^2/R_{\text{atom}}^2 \sim 10^{-8}$
Combination of Large Z and relativistic wf's offset by $10\,Z^2 \approx 10^5$, with overall suppression of $d_{\text{atom}} \sim 10^{-3} d_{\text{nucleus}}$

Best measurements in diamagnetic (atomic total angular momentum 0) ^{199}Hg constrain strong interaction $\not{T}$ competitive with neutron EDM.

A nuclear magnetic quadrupole moment is also $\not{T}$. This also produces an observable atomic EDM, yet with no screening Haxton+Henley PRL 51 1937 (1983)**, so it's more accurate to interpret experiments. (The total atomic angular momentum must be nonzero, so stray Larmor precession of 1000x greater μ makes experiments challenging.)**

Butler, Proc. R. Soc. A 476: 20200202 (2020)





Enhancement by octupole deformation

'Octupole deformation' produces low-lying parity doublet with strong E1 transition between them. This:

- **enhances mixing of opposite-parity states**
- **enhances the resulting Schiff moment because of the octupole and quadrupole deformations.**

In one model Flambaum Feldmeier PRC 101 015502 (2020) **in terms of strong $\not{T}$ constant η:**

$$S \approx 1 \times 10^{-4} \frac{J}{J+1} \beta_2 \beta_3^2 Z A^{\frac{2}{3}} \frac{\mathrm{keV}}{E^- - E^+} e\eta \mathrm{fm}^3$$

Result is 100-1000 x enhancement, in a sense restoring the full effect of the nuclear EDM, and in one case 10^4 or 10^5 enhancement going beyond (if a low-lying state is really the same J with opposite π).

$\not{T}$ in QCD and nucleon-nucleon interactions

$\mathcal{L}_{\not{CP}} = \theta_{QCD}\, \frac{g^2}{32\pi^2}\, F^{\mu\nu}_{\alpha} F^{*}_{\alpha\mu\nu}$

From the small neutron EDM, $\theta_{QCD} \lesssim 0.5x10^{-10}$

Peccei-Quinn mechanism drives θ_{QCD} small by a global U(1):
breaking the U(1) produces a 0^- axion with mass $\propto$ (symmetry-breaking scale)/(coupling).
Null experiments drive that scale high.

Other $\not{T}$ physics in the N-N potential is parameterized by
isoscalar, isovector, and isotensor terms, with a separate set for whether or not they break *P*.
(These can be related by the $1/N_c$ expansion Samart PRC 94 024001 (2016))

The QCD and effective nucleon-nucleon $\not{T}$ physics produces:

- **$\not{T}$ nuclear Schiff and magnetic quadrupole moments,**
- **$\not{T}$ asymmetries in polarized beam experiments (Simonius PRL 78 4161 (1997))**
- **$\not{T}$ asymmetries in polarized neutron experiments on polarized targets ($\lesssim 10^{-5}$ Huffman et al. PRC 55 2284 (1997), with plans to improve these at next-generation neutron sources enough to complement *n* and ^{199}Hg EDM experiments.)**

If one sees these asymmetries, they are from $\not{T}$: unlike decays, they are free of ‘final-state interaction’ false effects.

Nuclear nearest-level spacing and $\not{T}$

Bohr and Mottleson 2C-2:
Assume a Hamiltonian matrix with random values, the Gaussian Orthogonal Ensemble (GOE). Diagonalizing the Hamiltonian produces a statistical distribution of level spacings ϵ in terms of average spacing D (the "Wigner distribution")

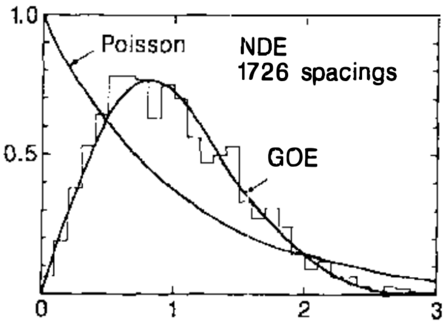
$$P(\epsilon) = \frac{\pi}{2D^2}\epsilon e^{-\frac{\pi}{4}\frac{\epsilon^2}{D^2}}$$

$$P(\epsilon) \overset{\epsilon\to 0}{\propto} \epsilon$$

This was for time-reversal invariant interactions. If you allow for $\not{T}$, you have unitary matrices instead, the Gaussian Unitary Ensemble (GUE) with twice as many elements, because they're complex. Then

$$P(\epsilon) \overset{\epsilon\to 0}{\propto} \epsilon^2$$

Bohigas+Weindenmüller Ann Rev Nucl Part Sci 38 421 1988



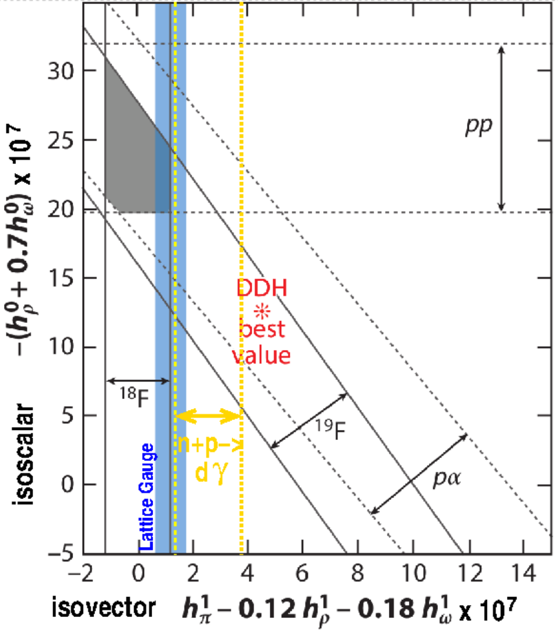
More sophisticated statistical measures extract an upper limit for the amount of $\not{T}$ in nuclear interactions $\alpha \lesssim 2\times 10^{-3}$ (J.B.French Ann. Phys. 181 235 (1988)). It's treated as an upper bound, since nuclear level spacings are not necessarily random ☺

Weak Neutral Current

Existence of Z^0 boson, spin-1 partner of $W^{\pm}$ and the photon, was a S.M. prediction.

Searched for in :

- **ν scattering (winner: Gargamelle)**
- **Atomic $\not{P}$ by mixing levels of opposite parity**
(1st answers came in small, creating concern for the S.M. prediction; now the best low-energy measurement of electron-quark weak neutral coupling)
- **A few years ago, "coherent" scattering of ν from nuclei is agreeing so far with SM cross-section**
- **parity violating nucleon-nucleon interaction, via γ asymmetries from decay of nuclear states: was in the race, but the effect was smaller than known $\rightarrow$**



Gardner Haxton Holstein ARNPS 2017+ Blythe PRL 121 242002 (2018)

Weak interaction between nucleons, $\not{P}$

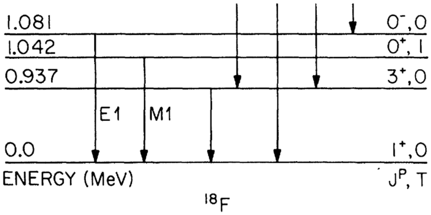
$W^{\pm}$, Z^0 (m=80.4, 91.2 GeV) are very short-ranged compared to mesons.

- **Parameterized by meson exchange (emitted weakly, absorbed strongly...)**
The isovector piece was long expected to be enhanced by the weak neutral current, but the 1/Nc expansion suppresses isovector/isoscalar by $\sin^2(\theta_W)/N_C \approx 1/12$ (Phillips et al. PRL 114 062301 (2015).
- **A formal EFT produces similar results.**
- **Isovector and isoscalar parts now considered measured.**
An isotensor part is interesting and inspiring proposals like $\vec{\gamma} + d \rightarrow n + p$

E.g. measuring N-N $\not{P}$ by mixing of ^{18}F 0^- and 0^+ states: much nuclear physics

- Observable is the circular polarization of the 1.081 MeV γ-ray, caused by E1 interference with the parity-violating M1, $-0.7 \pm 2.0 \times 10^{-3}$

Sensivity is enhanced by:



Barnes et al. PRL 40 840 (1977)

- The J^+; T= 0^-; 0 and 0^+; 1 states lie close together in energy, admixture $\propto \frac{\langle 0^- | \mathcal{O}_{\text{WeakNN}} | 0^+ \rangle}{\Delta E}$
- The E1 operator is isovector (except for a tiny correction from the long-wavelength approximation), so is suppressed by $\sim 10^{-4}$ between the T=0 states, so the parity-violating M1 competes better so the circular polarization is larger ☺

- A hard-to-calculate nuclear matrix element is needed to extract the weak N-N physics. (We noticed the 0^- state involves excitations of the *p* shell, which is quite complicated.) The same effective operator contributes, with known β-decay constants of proportionality, to the forbidden β decay of the isobaric analog 0^+; $T = 1$ state in ^{18}Ne.

Summarized in Haxton PRL 46 698 (1981) and the experimental paper before it Adelberger, Hoyle, Swanson, Lintig 695

- The experimental asymmetry measured was $\sim 10^{-5}$, while in n(p,d)γ was 3x10^{-8}

SM 2nd-order weak $\nu\nu\beta\beta$ vs $\not{SM}$ $0\nu\beta\beta$ decay

We've already seen *SM* $\beta\beta\nu\nu$ decay. 1st measured geochemically, then directly in very-low-background experiments. In $0\nu\beta\beta$ all energy is captured, a distinctive signature.

Kayser Journal of Physics: Conference Series 173 (2009) 012013;
Primakoff and Rosen 1959 Rep. Prog. Phys. 22 121

Particle physics for $0\nu\beta\beta$ to happen:

- Lepton number must not be conserved
- ν's have mass
- At least one of these two:
  a $\not{SM}$ interaction breaks lepton number; or,
  the ν has a "Majorana mass term" so
  part of $\nu = \bar{\nu}$

Much phenomenology happens then: e.g. diagonalizing the Dirac+Majorana mass matrix to find the actual masses naturally generates the light ν mass observed

To quantify dependence on ν mass, nuclear matrix elements need calculation

Suhonen Front. Phys. 5 art 55 p.1 (2017)

- 0^+ parent, progeny, $\nu\nu\beta\beta$ dominated by 1^+ intermediate states, GT transitions.
- $0\nu\beta\beta$ has contributions from forbidden operators and more spins, so $\nu\nu\beta\beta$ is not a complete benchmark for theory.
- A variety of approximate many-body answers vary by 2-4$\times$.

^{48}Ca is reachable now by more exact many-body methods

There are several approaches with detectors made out of the parent nuclei.

(Boehm+Vogel "Physics of massive ν's" crudely set non-rel $F(Z, E) \sim \frac{E}{p}\frac{2\pi\alpha Z}{1-e^{-2\pi\alpha Z}}$ (sort of ok for spectrum, poor for rates) to allow analytic phase space integrals $\sim E_0^{11}$ for $\nu\nu\beta\beta$ and $\sim E_0^5$ for $0\nu\beta\beta$)