I

INTERACTION OF ELECTRONS AND α-PARTICLES WITH MATTER

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INTERACTION OF ELECTRONS

Electrons penetrating matter lose energy and are deflected from their original course: they are scattered. Changes take place also in the matter that is penetrated, the constituent atoms are excited or ionized, and dissociation of molecules, changes in the lattice structure of crystals, changes in the conductivity, and many other secondary processes have been observed. These phenomena will be discussed here only if they give direct information about the energy of the electrons, as e.g. the ionization. Furthermore, we will restrict the discussion to electron energies in the range 10^4–10^7 eV, i.e. to the region of the radioactive β-emitters. For these energies, the deflexion of the electrons is due almost entirely to the elastic collisions with the atomic nuclei, while the energy loss, except that due to the bremsstrahlung, which is practically negligible, results from the interaction with the atomic electrons. Therefore it is possible to treat the two phenomena separately, though of course they always occur together. For positrons, the general behaviour is the same as for electrons. However, there are deviations which are mentioned at the corresponding places. A detailed review has been given by Bothe[1] and by Bethe and Ashkin[2], while the theoretical principles have also been discussed in detail by Sauter[3]. For general discussion of the theory see Mott and Massey[4].

§ 1. Elastic scattering of electrons by atomic nuclei

The elastic scattering of electrons passing through matter can be divided roughly into four classes: (1) Single scattering; (2) Plural scattering; (3) Multiple scattering; (4) Diffusion. If the thickness d of the layer is very small, $d \ll 1/\sigma N$, where σ is the cross-section and N the number of scattering atoms per cm^3, we have practically only single scattering, i.e. nearly all the scattered electrons are scattered by only one nucleus. It should be remembered, however, that the cross-section for the scattering of electrons by nuclei decreases very strongly with increasing scattering angle, so that the relation given above for the thickness of the layer shows a pronounced dependence on the angle of scattering Θ. For larger values of the thickness, $d \sim 1/\sigma N$, we get plural scattering, i.e. the probability that a given scattering angle is due to a number of

[1] W. Bothe, Handbuch der Physik **22/2** (Berlin, 1932) p. 1.
[2] H. A. Bethe and J. Ashkin, Experimental Nuclear Physics **1** (ed. E. Segrè; New York, 1953).
[3] F. Sauter, in: Kosmische Strahlung (ed. W. Heisenberg; Berlin, 1953).
[4] N. F. Mott, Proc. Roy. Soc. **A124** (1929) 425; N. F. Mott and H. S. W. Massey, Theory of Atomic Collisions (Oxford, 1948).

K. Siegbahn, Editor, Alpha-, Beta- and Gamma-Ray Spectroscopy

successive single scattering processes becomes appreciable. When the thickness becomes so large that the mean number n of scattering processes becomes larger than about 20, we speak of multiple scattering. The angular distribution $W(\Theta)$ of the scattered electrons is approximately gaussian as long as the mean scattering angle is smaller than about 20°. For still larger values of the thickness, $d \gg 1/\sigma N$, the angular distribution becomes of the form $W(\Theta) \propto \cos^2\Theta$. The mean angle of scattering $\bar{\Theta}$ then attains its maximum value $\Theta_{\max} \approx 33°$, and remains constant when the thickness increases still further ('normal' case, or 'complete diffusion'). The thickness for which the normal case is reached is called the 'normal thickness' d_n. Finally, electrons emerge from the foil also on the side of the incident beam. These electrons are either primary electrons which are deflected in the backward direction by single, plural or multiple scattering (back-scattering or back-diffusion), or secondary electrons which, however, are of no interest here. The number of back-scattered electrons reaches a saturation value for a definite thickness d_r, the 'thickness for saturation back-scattering', or 'back-diffusion' thickness.

1.1. Single scattering

The probability that an electron with kinetic energy E is scattered during the passage of a foil of thickness d and atomic number Z through an angle Θ into the solid angle $\mathrm{d}\Omega$ is given by

$$W(\Theta)\,\mathrm{d}\Omega = Nd \cdot \mathrm{d}\sigma(E, Z, \Theta)\,,$$

where N is the number of scattering atoms per cm^3. For the pure Coulomb field of a point charge without shielding we get, according to Mott[4]:

$$\mathrm{d}\sigma/\mathrm{d}\Omega = q_{\text{Mott}} = q_{\text{Ruth.}}\, R(E, Z, \Theta)\,.$$

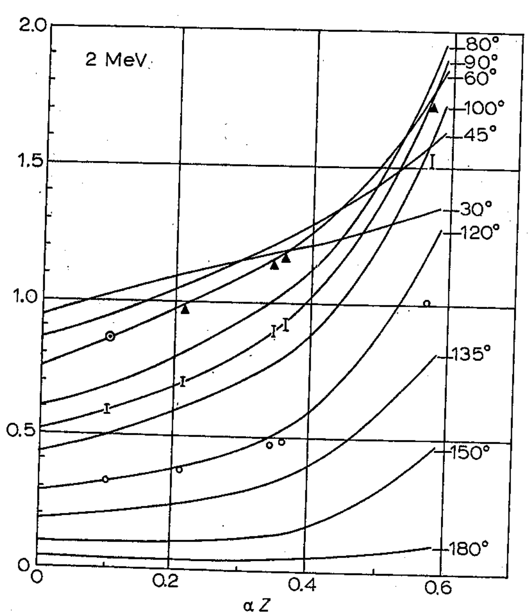


Fig. 1. The ratio $R = q_{\text{Mott}}/q_{\text{Ruth.}}$ as a function of αZ for various scattering angles and 2 MeV electrons. Experimental points from Paul and Reich[6]; the value for Al for 60° is fitted to the theoretical value

Here $q_{\text{Ruth.}}$ is the Rutherford cross-section

$$q_{\text{Ruth.}} = \frac{Z^2 r_0^2}{4 \sin^4 \frac{1}{2}\theta} \frac{1}{\varepsilon^2 (1 - 1/\varepsilon^2)^2}$$

$$\left(r_0 = e^2/mc^2 = \text{classical electron radius} = 2.8 \times 10^{-13}\ \text{cm},\ \varepsilon = \frac{E + mc^2}{mc^2} \right).$$

The factor $R(E, Z, \Theta)$ calculated by Mott cannot be expressed in analytic form. McKinley and Feshbach[5] expanded this factor in powers of $Z\alpha$ (α = fine structure constant), and the coefficients have been tabulated up to $(Z\alpha)^4$. R is different for electrons and positrons, and this difference increases with increasing energy and increasing angle of scattering and disappears in the non-relativistic limit ($R = 1$). R is shown in Fig. 1 for 2 MeV electrons and various scattering angles. The expression given above is a good representation of the experimental results for a large range of energy and scattering angle. However, the following deviations should be noted:

(1) For very small scattering angles the shielding of the Coulomb field of the nucleus by the atomic electrons is no longer negligible. The resulting deviations decrease with increasing energy. They have been calculated by Molière[7] on the basis of the Thomas–Fermi model, and are shown in Fig. 2, where Θ_0 is the 'shielding angle'

$$\Theta_0 = \frac{0.47 Z^{\frac{1}{3}}}{p/mc} \sqrt{1.13 + 3.76\,(Z\alpha/\beta)^2} \qquad [\text{degree}].$$

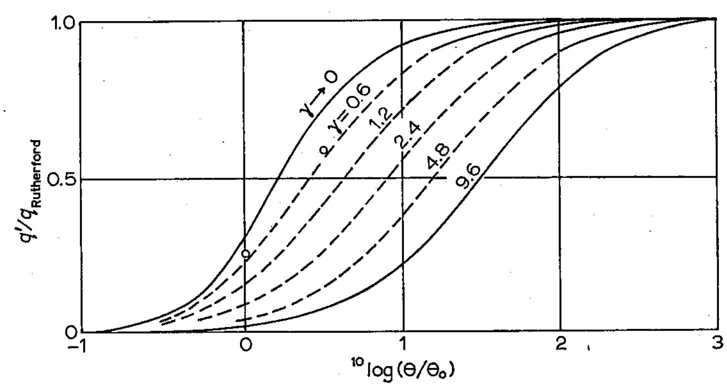


Fig. 2. Influence of the shielding of the Coulomb field by the atomic electrons on the scattering cross-section for various values of the parameter $\gamma = \alpha Z/\beta$; Θ_0 = shielding angle; q' = cross-section with shielding

(2) For high energies and large scattering angles deviations occur because of the finite extension of the nucleus*. According to calculations of Freese and Hain[8], the

* Cf. the experiments at 17 MeV by E. M. Lyman, A. D. Hanson and M. B. Scott, Phys. Rev. **84** (1951) 626.

[5] W. A. McKinley and H. Feshbach, Phys. Rev. **74** (1948) 1759.

resulting deviations are already appreciable for energies of about 1 MeV, cf. the measurements of Paul and Reich[6] in Fig. 1.

(3) For high energies we must finally take into account the quantum electrodynamic corrections, which arise from the emission of very soft light quanta which cannot be detected experimentally[9].

1.2. Plural scattering

Not many quantitative theoretical or experimental results concerning the plural scattering are available. In principle, the plural scattering is contained in Molière's theory of multiple scattering[10], but the resulting expressions are useful only if the plural scattering gives not more than a small correction to the multiple scattering.

1.3. Multiple scattering

The multiple scattering has been investigated theoretically by Bothe[1] and by Williams[11], and more recently in particular by Molière[10] and by Snyder and Scott[12]. For small values of the mean angle of scattering ($\bar{\Theta} < 20°$) in the region of multiple scattering (mean number of collisions $n > 20$, cf. Fig. 3), Molière obtains relatively simple expressions for the angular distribution, which are very well verified experimentally (cf. below). According to Molière, the probability that an electron passing through a

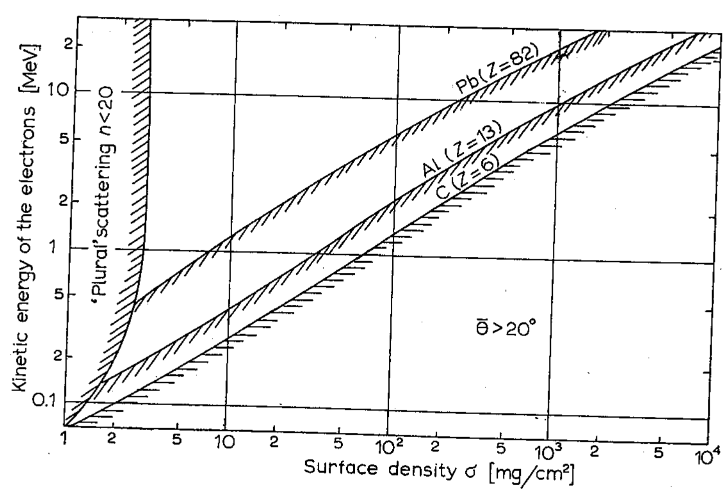


Fig. 3. Approximate range of validity of Molière's theory

[6] W. Paul and H. Reich, Z. Phys. **131** (1952) 326.
[7] G. Molière, Z. Naturf. **2a** (1947) 133.
[8] E. Freese and K. Hain, Z. Naturf. **9a** (1954) 456.
[9] J. Schwinger, Phys. Rev. **75** (1949) 898.
[10] G. Molière, Z. Naturf. **3a** (1948) 78.
[11] E. J. Williams, Proc. Roy. Soc. **A169** (1950) 531.
[12] H. S. Snyder and W. T. Scott, Phys. Rev. **76** (1949) 220.

foil of thickness d is scattered through an angle Θ into the solid angle $\mathrm{d}\Omega \approx 2\pi\Theta\,\mathrm{d}\Theta$ is given by

$$W(\Theta)\,2\pi\Theta\,\mathrm{d}\Theta = [2\exp(-\vartheta^2) + F_1(\vartheta)/B + F_2(\vartheta)/B^2 + \ldots]\,\vartheta\,\mathrm{d}\vartheta\,,$$

in which

$$\vartheta = \Theta/\chi_c\sqrt{B}\,; \qquad \chi_c = \frac{44.8Z}{\left(\dfrac{E+mc^2}{mc^2} - \dfrac{mc^2}{E+mc^2}\right)}\sqrt{\frac{\sigma}{A}} \qquad [\text{degree}]\,.$$

When we project this angular distribution on a plane parallel to the direction of the incident electron, we get for the 'projected angular distribution' as a function of the projected angle φ:

$$W(\Phi)\,\mathrm{d}\Phi = [\pi^{-\frac{1}{2}}\exp(-\,\varphi^2) + f_1(\varphi)/B + f_2(\varphi)/B^2 + \ldots]\,\mathrm{d}\varphi\,,$$

where $\varphi = \Phi/\chi_c\sqrt{B}$. Z is the atomic number and A the atomic weight of the scattering atoms, E and m are the kinetic energy and the mass of the electron, and σ, the surface density in g/cm^2, is a measure for the thickness of the foil. B depends slightly on the mean number of collisions, n, which according to Molière is given by

$$^{10}\log n = 8.215 + {}^{10}\log\left[\frac{\sigma}{AZ^{\frac{4}{3}}}\;\frac{\gamma^2}{1.13 + 3.76\gamma^2}\right],$$

where $\gamma = \alpha Z/\beta$. B is given in Table 1 for various values of n.

TABLE 1

$^{10}\log n$	1	2	3	4	5	6	7	8	9
B	3.36	6.29	8.93	11.49	13.99	16.46	18.90	21.32	23.71

The functions F_1, F_2 and f_1, f_2 have been calculated and tabulated by Molière, cf. Table 2.

Thus for not too small values of B the angular distribution of the scattered electrons is essentially a gaussian distribution; the higher terms are corrections arising from the single and plural scattering. Finally, the mean value of the scattering angle Θ is given by

$$(2/\pi)\,\bar{\Theta} = \bar{\Phi} = \chi_c\sqrt{B}\,(1 + 0.982/B - 0.117/B^2 + \ldots)\,.$$

For quick orientation, the values of $\chi_c\sqrt{B}$ ($\approx \bar{\Theta}$) for aluminum ($Z = 13$) and gold ($Z = 79$) are shown in Fig. 4 as a function of energy and thickness.

The multiple scattering has been investigated experimentally by Kulchitsky and Latyshev[13] for a large number of elements with 2 MeV electrons, and by Hanson *et al.*[14]

13 L. A. Kulchitsky and G. D. Latyshev, Phys. Rev. **61** (1942) 254.
14 A. O. Hanson, L. H. Lanzl, E. M. Lyman and M. B. Scott, Phys. Rev. **84** (1951) 634.

TABLE 2

$F_1(\vartheta)$	$F_2(\vartheta)$	ϑ, φ	$f_1(\varphi)$	$f_2(\varphi)$
+ 0,8456	+ 2,49	0,0	+ 0,0206	+ 0,416
+ 0.700	+ 2.07	0.2	− 0.0246	+ 0.299
+ 0.343	+ 1.05	0.4	− 0.1336	+ 0.019
− 0.073	− 0.003	0.6	− 0.2440	− 0.229
− 0.396	− 0.606	0.8	− 0.2953	− 0.292
− 0.528	− 0.636	1	− 0.2630	− 0.174
− 0.477	− 0.305	1.2	− 0.1622	+ 0.010
− 0.318	+ 0.052	1.4	− 0.0423	+ 0.138
− 0.147	+ 0.243	1.6	+ 0.0609	+ 0.146
0.000	+ 0.238	1.8	+ 0.1274	+ 0.094
+ 0.080	+ 0.131	2	+ 0.147	+ 0.045
+ 0.106	+ 0.020	2.2	+ 0.142	− 0.049
+ 0.101	− 0.046	2.4	+ 0.1225	− 0.071
+ 0.082	− 0.064	2.6	+ 0.100	− 0.064
+ 0.062	− 0.055	2.8	+ 0.078	− 0.043
+ 0.045	− 0.036	3	+ 0.059	− 0.024
+ 0.033	− 0.019	3.2	+ 0.045	− 0.010
+ 0.0206	+ 0.0052	3.5	+ 0.0316	+ 0.001
+ 0.0105	+ 0.0011	4	+ 0.0194	+ 0.006
$+ 3.82\times10^{-3}$	$+ 0.836\times10^{-3}$	5	$+ 9.14\times10^{-3}$	$+ 1.98\times10^{-3}$
$+ 1.74\times10^{-3}$	$+ 0.345\times10^{-3}$	6	$+ 5.06\times10^{-3}$	$+ 0.928\times10^{-3}$
$+ 0.91\times10^{-3}$	$+ 0.157\times10^{-3}$	7	$+ 3.12\times10^{-3}$	$+ 0.482\times10^{-3}$

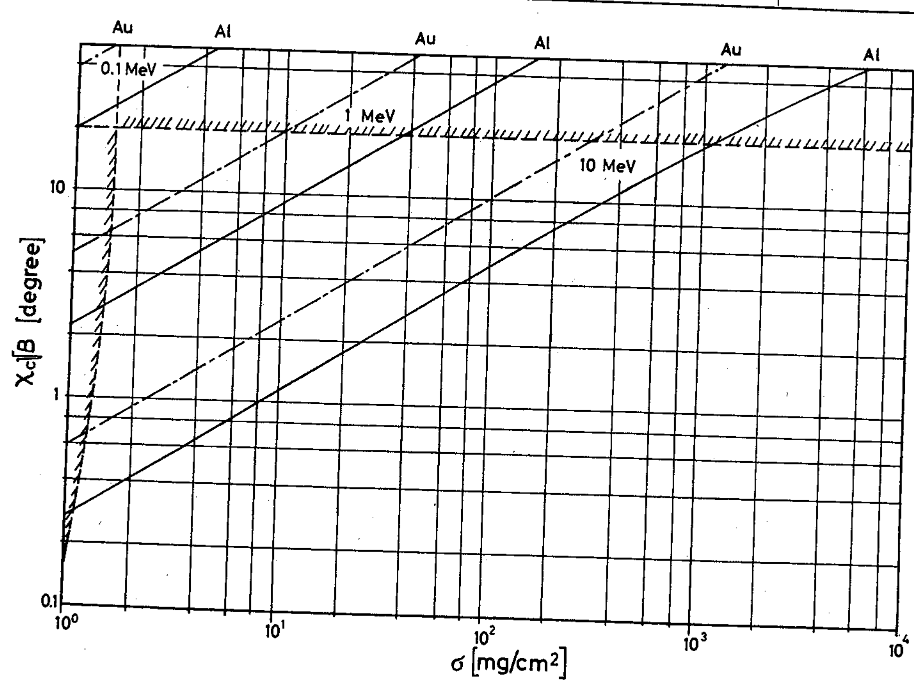


Fig. 4. The parameter $\chi_c\sqrt{B}\,(\approx\overline{\Theta})$ in Molière's theory for Al and Au as a function of energy and thickness. The boundaries of the theory are indicated on the left side and at the top

for Be and Au with 15.7 MeV electrons. Excellent agreement was obtained with the above expressions. In Fig. 5 the experimental results of Hanson *et al.* are shown.

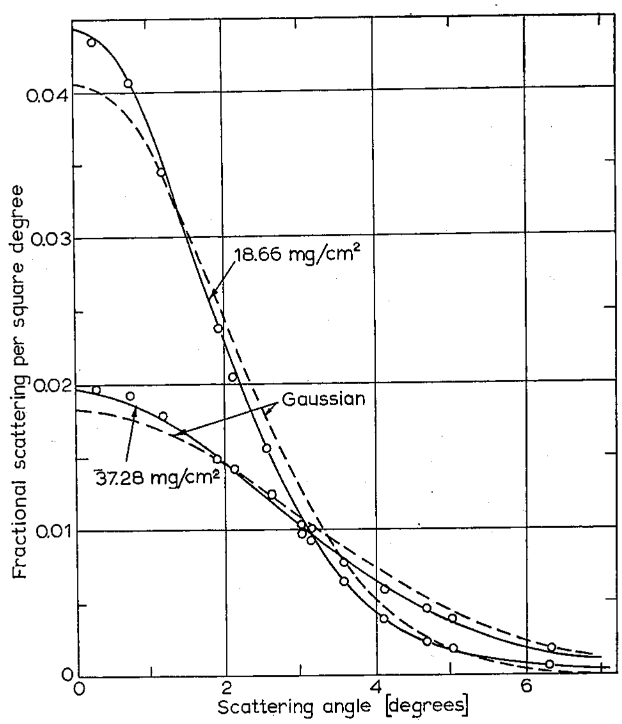


Fig. 5. Angular distribution[14] for multiple scattering of 15.7 MeV electrons passing through gold foils (18.66 mg/cm^2; 37.28 mg/cm^2)

1.4. Scattering in thick foils. Diffusion

For thick foils, theoretical calculations are very difficult. Only for the case of 'complete diffusion' it is possible to give an analytical expression as shown by Bethe *et al.*[15]. They find

$$W(\Theta)/W(0) = (0.717 + \cos\Theta)\cos\Theta\ .$$

Measurements by Frank[16] show how the angular distribution changes from multiple scattering to complete diffusion. The results are given in Fig. 6. Frank finds also that the energy distribution varies very little with angle. The normal thickness d_n above which the distribution changes no more, is shown in Fig. 7 for normal incidence of the electrons.

1.5. Back-scattering and diffusion

For back-scattering by thick foils there exist no theoretical calculations. Only the case

[15] H. A. Bethe, M. E. Rose and L. P. Smith, Proc. Amer. Phil. Soc. **78** (1938) 573.
[16] H. Frank, Z. Naturf. **14a** (1959) 247.
[17] W. Wilson, Proc. Roy. Soc. A**87** (1912) 100, 310.

of an infinite medium has been treated by Bothe [20] and Spencer [21]. The change from back-scattering to back-diffusion has been studied by Frank [16]; results are shown in

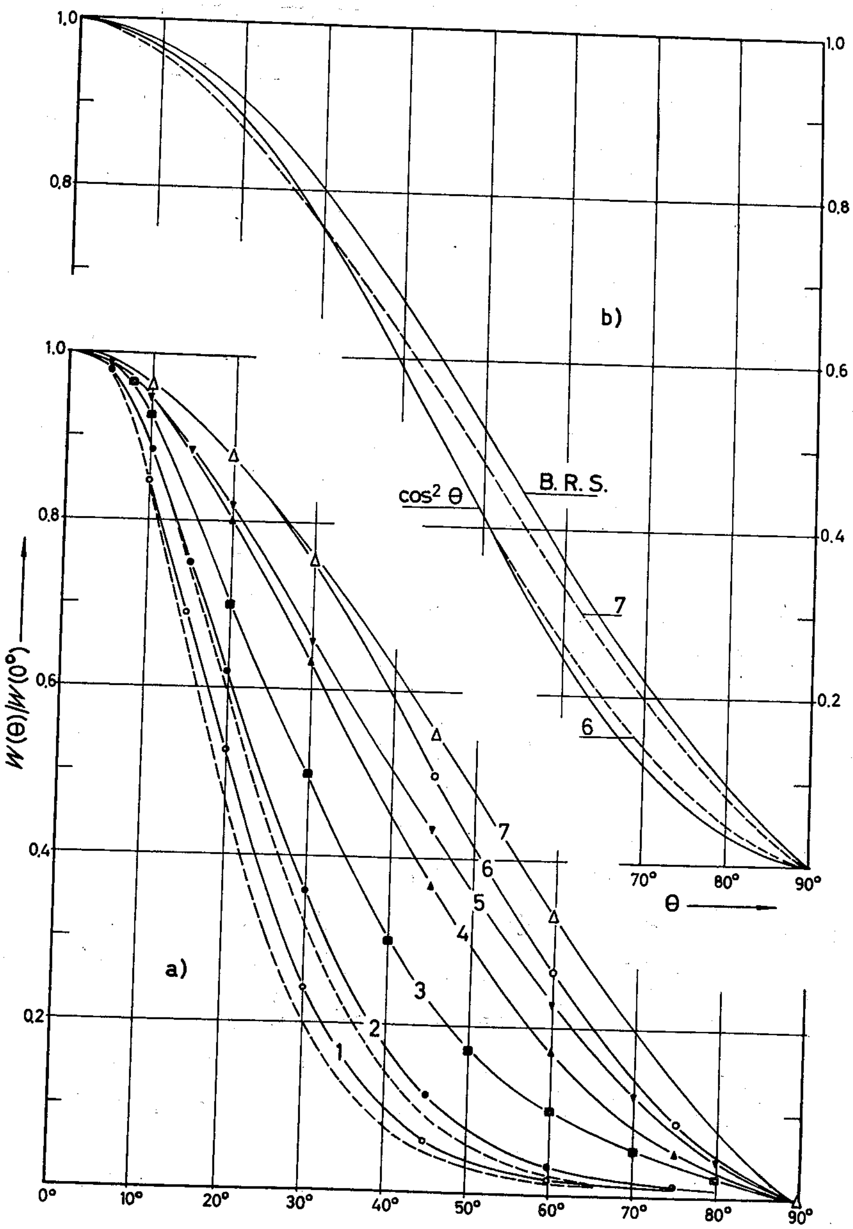


Fig. 6. Relative angular distribution for various thicknesses and materials. The dashed curves are according to Molière's theory [10], the curve noted B.R.S. shows the distribution of Bethe, Rose and Smith [15]. The numbered curves are measured by Frank [16]:

No.	1	2	3	4	5	6		7
	Cu	Al	Pb	Al	Cu	Pb	Al	Pb
mg/cm²	51.1	132	50.2	260	180	110	545	463

Fig. 8. The back-diffusion thickness d_r as a function of the maximum energy of the β-spectrum is shown in Fig. 9. In Fig. 10 the back-diffusion coefficient p which gives the ratio of back-scattered to incident electrons at saturation, is shown as a function of the atomic number both for parallel and for diffuse incidence.

Finally, in Fig. 11, the experimental data of Bothe[20] on the energy distribution of the back-scattered electrons are shown. Bothe's theory[20] shows that p is independent

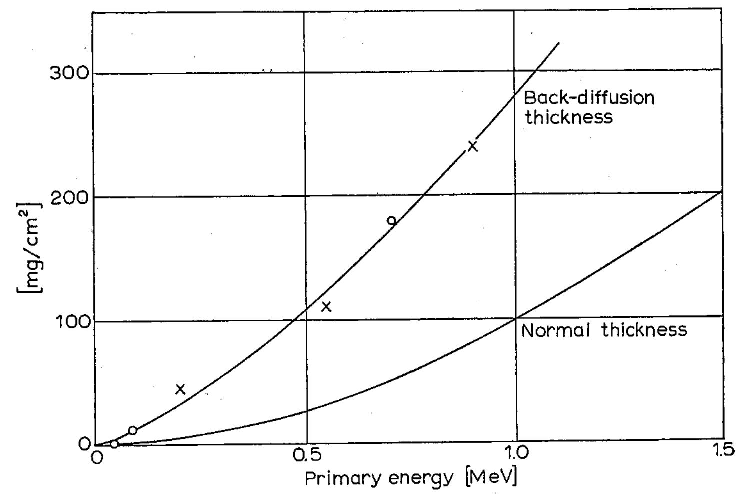


Fig. 7. Normal thickness and back-diffusion thickness for normal incidence. Experimental points are for Al (refs. 17–19)

of the primary energy. The back-scattering coefficient for positrons is different from that for electrons[28], and this difference between electrons and positrons is due to the fact that for large scattering angles the single scattering is different for electrons and positrons (cf. above). We have $p^-/p^+ = 1.3$, independent of Z (measured from Be to Pb).

§ 2. Energy loss

When electrons of a definite energy pass through a foil of matter, one observes that their energy is decreased, as is shown e.g. in Fig. 12. The energy spectrum becomes

[18] B. F. J. Schonland, Proc. Roy. Soc. **A108** (1925) 187.
[19] P. Lenard, Quantitatives über Kathodenstrahlen (Heidelberg, 1928).
[20] W. Bothe, Z. Naturf. **4a** (1949) 542.
[21] L. V. Spencer, Phys. Rev. **98** (1955) 1597.
[22] L. Yaffe and K. Justus, J. Chem. Soc. **5** (1949) 341.
[23] A. F. Kovaric, Phil. Mag. **20** (1910) 849.
[24] H. W. Schmidt, Ann. Phys. **23** (1907) 677.
[25] P. Palluel, Comptes Rendus **224** (1947) 1492, 1551.
[26] J. G. Trump and R. J. van de Graaff. Phys. Rev. **75** (1949) 44.
[27] H. V. Neher, Phys. Rev. **37** (1931) 655.
[28] R. R. Wilson, Phys. Rev. **60** (1941) 749.

broader and unsymmetrical. Thus a well-defined energy loss does not exist, one can define only a mean energy loss $\overline{\Delta E}$, or a probable energy loss, ΔE_W which is equal to the maximum of the energy distribution curve. This energy loss is due to the inelastic collisions of the electrons with the atomic electrons, by which the atoms are excited or ionized, and to the emission of bremsstrahlung in the Coulomb field of the nucleus.

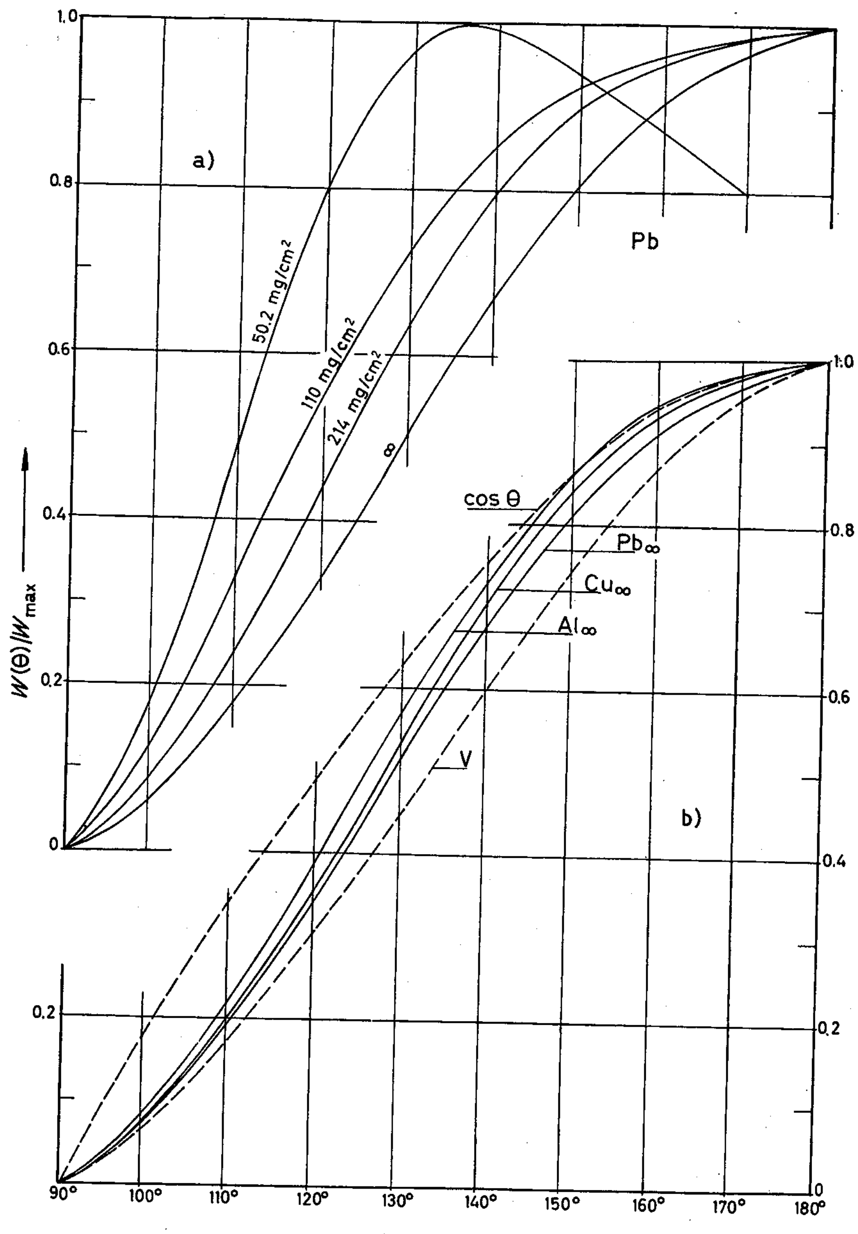


Fig. 8. Relative angular distribution of back-scattered electrons for normal incidence: a) Pb foils of various thicknesses, b) Saturation for Al, Cu, and Pb. The curve called 'V' is the same as No. 6 from Fig. 6

2.1. Energy loss due to inelastic collisions

The interaction of the incident electrons with the atomic electrons in the foil is characterized by the fact that the energy transferred to the atoms per collision is very small. Even for very high primary energies excitation is more probable than ionization, and the resulting secondary electrons have a mean kinetic energy of only a few eV. The total energy loss after passage through a foil of thickness x is therefore the result of a very large number of small energy losses. The theory was developed mainly by

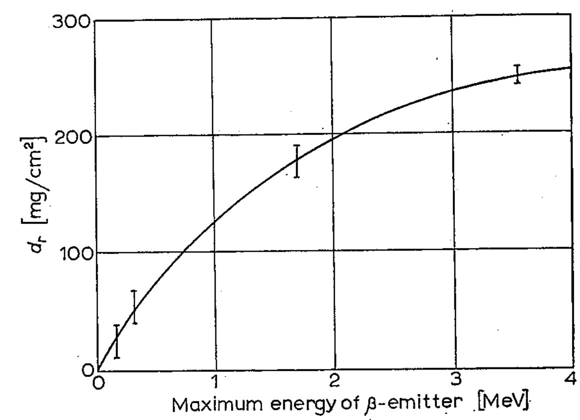
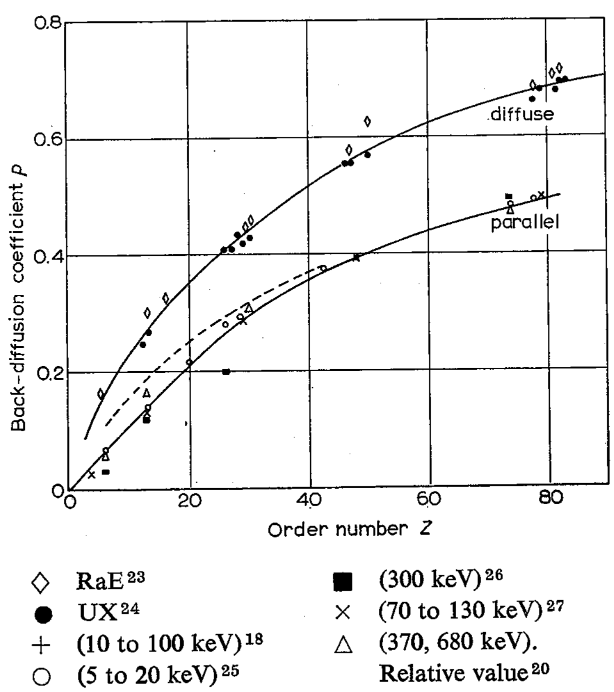


Fig. 9. Thickness for saturation back-scattering, d_r, for diffuse angular distribution of the incident electrons, measured with P^{32}, Rh^{106}, I^{131}, S^{35}, Co^{60} and C^{14} (cf. ref. 22)



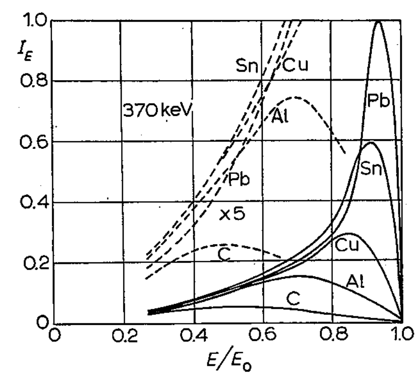


Fig. 11

Fig. 10 (left). Back-diffusion coefficient (saturation) for normal incidence, and for diffuse angular distribution of the incident beam